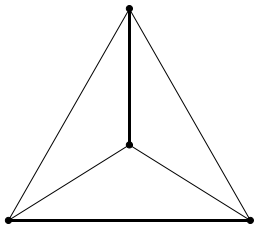
1.2 Season 1 Episode 2, 09/27/2015

1. [Based on Mandelbrot 2013] A marker is placed on each vertex (large dot) in the diagram shown to the right.

   Each of the four markers is red on one side and black on the reverse side; to begin all the red sides are facing up. On a move you swap any two markers and flip them both over. Determine with justification the least number of moves needed to return each marker to its starting vertex, but have them all black side up instead?

2. In the left-hand side diagram shown below, a castle is surrounded by a rectangular moat, which is of uniform width 12 feet. The problem is to get across the moat to the castle from the dry land on the other side, without being able to use the drawbridge. All you have to work with are two rectangular planks, whose lengths are 11 feet and 11 feet, 9 inches. Find a way to get across.

3. The right-hand diagram shown above provides a hint to dissect a 1×3 rectangle into four pieces that can be reassembled into a square. Can you explain the details of this method?

4. [MPG 2015] Solve the equation $\sqrt{x + 2015} = x^2 - 2015$ for x.

5. [China TST 2007, by Zuming Feng] Let k be a given positive integer greater than 1. An k-digit integer $a_1 a_{k-1} \ldots a_k$ is called *parity-monotonic* if for every integer i with $1 \leq i \leq k-1$,

$$\begin{cases} a_i > a_{i+1} & \text{if } a_i \text{ is odd,} \\ a_i < a_{i+1} & \text{if } a_i \text{ is even.} \end{cases}$$

   How many parity-monotonic integers are there?