

Lectures on Challenging Mathematics

MO1M1

Elements of Olympiad Mathematics Module 1

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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Contents

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1 Algebra	1
1.1 Algebra practice set 1	1
1.2 Polynomials and their roots	2
1.3 The first look at the AM-GM inequality	4
1.4 Algebra practice set 2	5
1.5 Computations with symmetric polynomials	6
1.6 A touch on weighted AM-GM	7
1.7 Algebra practice set 3	8
1.8 Introduction to functional properties (part 1)	9
1.9 The first look at the Cauchy-Schwarz inequality (part 1)	11
1.10 A quick look at the circle of Apollonius	12
1.11 Algebra practice set 4	13
1.12 Introduction to functional properties (part 2)	14
1.13 The first look at the Cauchy-Schwarz inequality (part 2)	15
1.14 Algebra practice set 5	16
1.15 Basic properties of polynomials	17
1.16 Inequality practice set 1	18
1.17 Algebra practice set 6	19
1.18 Introduction to functional properties (part 3)	20
1.19 Inequality practice set 2	21
1.20 Integer roots of quadratic equations	22
2 Combinatorics	23
2.1 Challenges in enumerative counting (part 1)	23
2.2 Brain teasers and mathematical reasoning (part 1)	24
2.3 Mathematical arguments – The first tour of the Pigeonhole principle	25
2.4 Challenges in enumerative counting (part 2)	26
2.5 Brain teasers and mathematical reasoning (part 2)	27
2.6 Mathematical arguments – Proof by contradiction	28
2.7 Challenges in enumerative counting (part 3)	29
2.8 Brain teasers and mathematical reasoning (part 3)	30
2.9 Mathematical arguments – The first tour of mathematical induction	31

2.10	Mathematical arguments – The second tour of the Pigeonhole principle	32
2.11	Challenges in enumerative counting (part 4)	34
2.12	Brain teasers and mathematical reasoning (part 4)	35
2.13	Mathematical arguments – The second tour of mathematical induction	36
2.14	Challenges in enumerative counting (part 5)	37
2.15	Brain teasers and mathematical reasoning (part 5)	38
2.16	Mathematical arguments – Proof by contradiction and Pigeonhole principle	39
2.17	Challenges in enumerative counting (part 6)	40
2.18	Brain teasers and mathematical reasoning (part 6)	41
2.19	Mathematical arguments – Parity argument	42
2.20	Mathematical arguments – Coloring	43
2.21	Challenges in enumerative counting (part 7)	44
2.22	Brain teasers and mathematical reasoning (part 7)	45
2.23	Mathematical arguments – The well ordering principle	46
2.24	Challenges in enumerative counting (part 8)	47
2.25	Brain teasers and mathematical reasoning (part 8)	48
3	More topics in Combinatorics	49
3.1	Mathematical arguments – Coloring and assigning weights (part 1)	49
3.2	Mathematical arguments – Coloring and assigning weights (part 2)	51
3.3	Math reasoning gems from AMC10A/12A 2016	52
3.4	Math reasoning gems from AMC10B/12B 2016	53
3.5	Math reasoning gems from AIME 2016	54
4	Practice tests	55
4.1	Advanced level problems from EMC ² 2010 (part 1)	55
4.2	Advanced level problems from EMC ² 2011 (part 1)	57
4.3	Advanced level problems from EMC ² 2010 (part 2)	58
4.4	Advanced level problems from EMC ² 2011 (part 2)	59
4.5	Advanced level problems from EMC ² 2010 and 2011	61