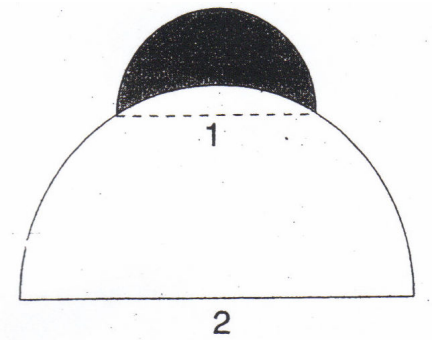
Geometry Part One
VI Circle

1 Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that spot can reach?

(A) $\frac{2}{3}\pi$ **(B)** 2π **(C)** $\frac{5}{2}\pi$ **(D)** $\frac{8}{3}\pi$ **(E)** 3π

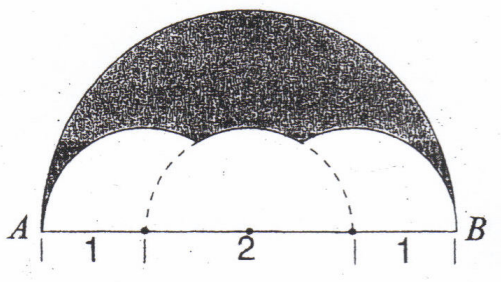
2 A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi-\frac{\sqrt{3}}{4}$ **(B)** $\frac{\sqrt{3}}{4}-\frac{1}{12}\pi$ **(C)** $\frac{\sqrt{3}}{4}-\frac{1}{24}\pi$

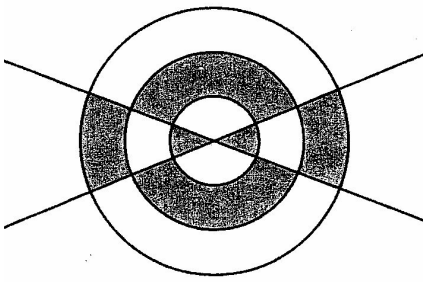
(D) $\frac{\sqrt{3}}{4}+\frac{1}{24}\pi$ **(E)** $\frac{\sqrt{3}}{4}+\frac{1}{12}\pi$

3 Three semicircles of radius 1 are constructed on Diameter $\overline{AB}$ of a semicircle of radius 2. The centers of the small semicircles divide $\overline{AB}$ into four line segments of equal lengths, as shown. What is the area of the shaded region that lies within the large semicircle but outside the smaller semicircles?



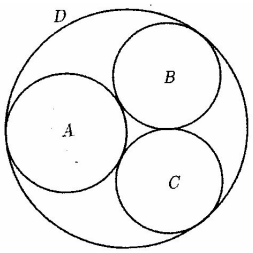
(A) $\pi-\sqrt{3}$ **(B)** $\pi-\sqrt{2}$ **(C)** $\frac{\pi+\sqrt{2}}{2}$ **(D)** $\frac{\pi+\sqrt{3}}{2}$ **(E)** $\frac{7}{6}\pi-\frac{\sqrt{3}}{2}$

4 Two distinct lines pass through the center of three oncentric circles of radii 3, 2, and 1. The area of the shaded region in the diagram is 8/13 of the area of the unshaded region. What is the radian measure of the acute angle formed by the two lines? (Note: π radians is 180 degrees.)



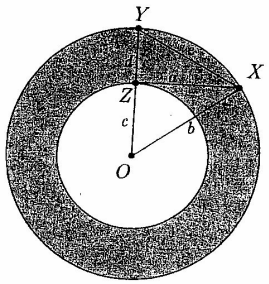
(A) $\frac{\pi}{8}$ **(B)** $\frac{\pi}{7}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{5}$ **(E)** $\frac{\pi}{4}$

5 Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Circles B and C are congruent. Circle A has radius 1 and passes through the center of D. What is the radius of circle B?



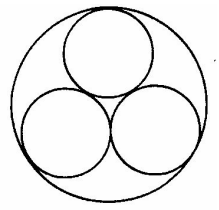
(A) $\frac{2}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{8}{9}$ **(E)** $\frac{1+\sqrt{3}}{3}$

6 An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b > c$. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?



(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2

7 Three circles of radius 1 are externally tangent to each other and internally tangent to a larger circle. What is the radius of the larger circle?



(A) $\frac{2+\sqrt{6}}{3}$ **(B)** 2 **(C)** $\frac{2+3\sqrt{2}}{3}$ **(D)** $\frac{3+2\sqrt{3}}{3}$ **(E)** $\frac{3+\sqrt{3}}{2}$

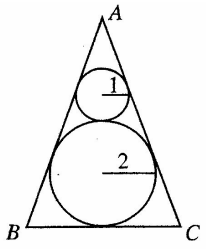
8 A triangle with sides 5, 12, 13 has both an inscribed and a circumscribed circle. What is the distance between the centers of those circles?

(A) $\frac{3\sqrt{5}}{2}$ **(B)** $\frac{7}{2}$ **(C)** $\sqrt{15}$ **(D)** $\frac{\sqrt{65}}{2}$ **(E)** $\frac{9}{2}$

9 In ΔABC we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D is on the circumscribed circle of the triangle so that $\overline{AD}$ bisects $\angle BAC$. What is the value of AD/CD?

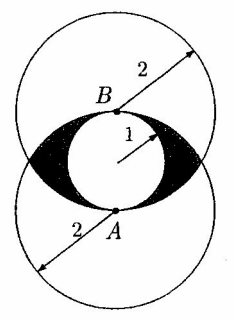
(A) $\frac{9}{8}$ **(B)** $\frac{5}{3}$ **(C)** 2 **(D)** $\frac{17}{7}$ **(E)** $\frac{5}{2}$

10 A circle of radius 1 is tangent to a circle of radius 2. The sides of ΔABC are tangent to the circles as shown, and the sides $\overline{AB}$ and $\overline{AC}$ are congruent. What is the area of ΔABC?



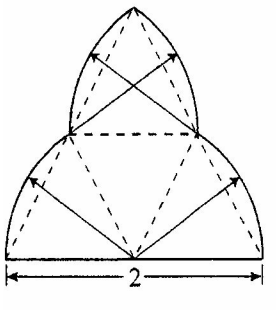
(A) $\frac{35}{2}$ **(B)** $15\sqrt{2}$ **(C)** $\frac{64}{3}$ **(D)** $16\sqrt{2}$ **(E)** 24

11 A circle of radius 1 is internally tangent to two circles of radius 2 at points A and B, where AB is a diameter of the smaller circle. What is the area of the region, shaded in the figure, that is outside the smaller circle and inside each of the two larger circles?



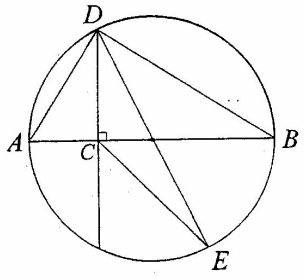
(A) $\frac{5}{3}\pi - 3\sqrt{2}$ **(B)** $\frac{5}{3}\pi - 2\sqrt{3}$ **(C)** $\frac{8}{3}\pi - 3\sqrt{3}$
(D) $\frac{8}{3}\pi - 3\sqrt{2}$ **(E)** $\frac{8}{3}\pi - 2\sqrt{3}$

12 The figure shown is a *trefoil* and is constructed by drawing circular sectors about sides of the congruent equilateral triangles. What is the area of a trefoil whose horizontal base has length 2?



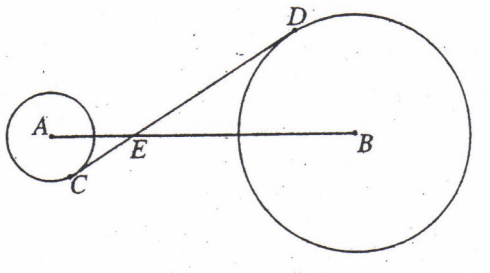
(A) $\frac{1}{3}\pi + \frac{\sqrt{3}}{2}$ **(B)** $\frac{2}{3}\pi$ **(C)** $\frac{2}{3}\pi + \frac{\sqrt{3}}{4}$
(D) $\frac{2}{3}\pi + \frac{\sqrt{3}}{3}$ **(E)** $\frac{2}{3}\pi + \frac{\sqrt{3}}{2}$

13 Let $\overline{AB}$ be a diameter of a circle and C be a point on $\overline{AB}$ with $2\,AC = BC$. Let D and E be points on the circle such that $\overline{DC} \perp \overline{AB}$ and $\overline{DE}$ is a second diameter. What is the ratio of the area of ΔDCE to the area of ΔABD?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

14 Circles with centers A and B have radii 3 and 8, respectively. A common internal tangent touches the circles at C and D, as shown. Lines AB and CD intersect at E, and $AE = 5$. What is CD?

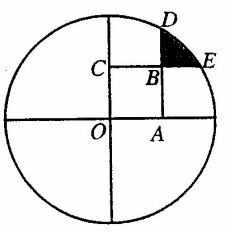


(A) 13 **(B)** $\frac{44}{3}$ **(C)** $\sqrt{221}$ **(D)** $\sqrt{255}$ **(E)** $\frac{55}{3}$

15 A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of this circle?

(A) 2π **(B)** $\frac{5\pi}{2}$ **(C)** $\frac{81\pi}{32}$ **(D)** 3π **(E)** $\frac{7\pi}{2}$

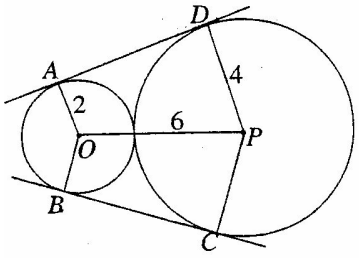
16 A circle of radius 2 is centered at *O*. Square *OABC* has side length 1. Sides $\overline{AB}$ and $\overline{CD}$ are extended past *B* to meet the circle at *D* and *E*, respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting *D* and *E*?



(A) $\frac{\pi}{3}+1-\sqrt{3}$ **(B)** $\frac{\pi}{2}(2-\sqrt{3})$ **(C)** $\pi(2-\sqrt{3})$

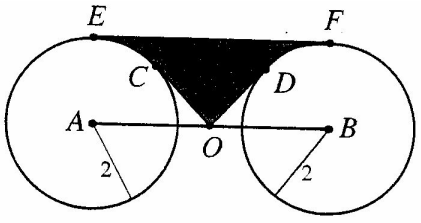
(D) $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$ **(E)** $\frac{\pi}{3}-1+\sqrt{3}$

17 Circles with centers at *O* and *P* have radii 2 and 4, respectively, and are externally tangent. Points *A* and *B* on the circle with center *O* and points *C* and *D* on the circle with center *P* are such that $\overline{AD}$ and $\overline{BC}$ are common external tangents to the circles. What is the area of the concave hexagon *AOBCPD*?



(A) $18\sqrt{3}$ **(B)** $24\sqrt{2}$ **(C)** 36 **(D)** $24\sqrt{3}$ **(E)** $32\sqrt{2}$

18 Circles centered at *A* and *B* each have radius 2, as shown. Point *O* is the midpoint of $\overline{AB}$, and $OA = 2\sqrt{2}$. Segments *OC* and *OD* are tangent to the circles centered at *A* and *B*, respectively, and $\overline{EF}$ is a common tangent. What is the area of the shaded region *ECODF*?

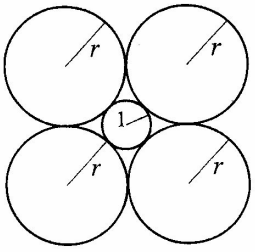


(A) $\frac{8\sqrt{2}}{3}$ **(B)** $8\sqrt{2}-4-\pi$ **(C)** $4\sqrt{2}$ **(D)** $4\sqrt{2}+\frac{\pi}{8}$ **(E)** $8\sqrt{2}-2-\frac{\pi}{2}$

19 Two circles of radius 2 are centered at (2, 0) and at (0, 2). What is the area of the intersection of the interiors of the two circles?

(A) $\pi-2$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\sqrt{3}\pi}{3}$ **(D)** $2(\pi-2)$ **(E)** π

20 A circle of radius 1 is surrounded by 4 circles of radius r as shown. What is r?



(A) $\sqrt{2}$ **(B)** $1+\sqrt{2}$ **(C)** $\sqrt{6}$ **(D)**3 **(E)** $2+\sqrt{2}$