

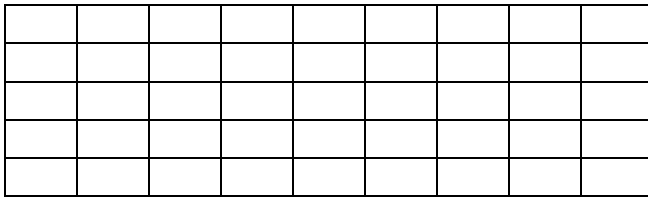
## A Mathematical Model for Counting

1 How many rectangles are in the figures?

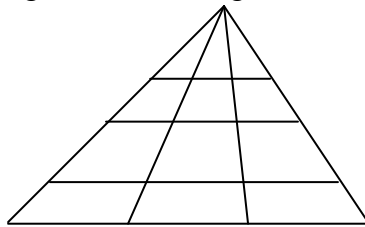
(a)



(b)

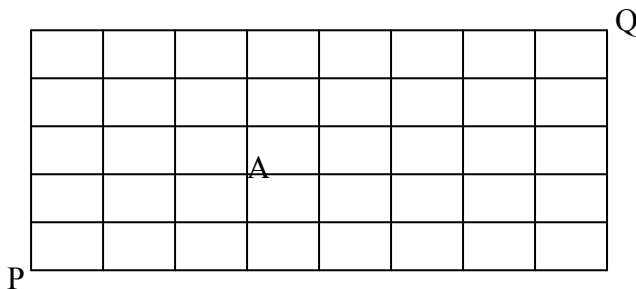


2 How many triangles are in the figure?



3 A group of six girls and four boys are lined up in single file. In how many ways can they be arranged, if each girl is to be in front of all shorter girls, and each boy is to be in front of all shorter boys?

4 The grid lines below represent streets in an 8-block by 5-block area. How many different 13-block routes are there (a) from P to Q? (b) from P to Q through A?

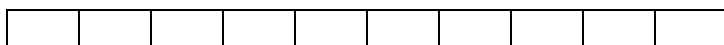


5 Let  $n = \overline{abc}$  be a 3-digit number.

(a) How many numbers  $n$  are there with  $a > b > c$ ?

(b) How many numbers  $n$  are there with  $a < b < c$ ?

- 6 (a) How many positive integer solutions does the indefinite equation  $x + y + z = 10$  have?
- (b) How many non-negative integer solutions does the indefinite equation  $w + x + y + z = 10$  have?
- (c) How many positive integer solutions does the linear inequality  $x + y < n$  have, where  $n$  is a positive integer and  $n \geq 3$ .
- (d) How many positive integer solutions does the linear inequality  $x + y \leq n$  have, where  $n$  is a positive integer and  $n \geq 3$ .
- (e) How many non-negative integer solutions does the linear inequality  $x + y < n$  have, where  $n$  is a positive integer and  $n \geq 3$ .
- (f) How many non-negative integer solutions does the linear inequality  $x + y \leq n$  have, where  $n$  is a positive integer and  $n \geq 3$ .
- (g) How many non-negative integer solutions does the linear inequality  $x + y + z \leq n$  have, where  $n$  is a non-negative integer?
- 7 How many ordered triples  $(a, b, c)$  are there such that  $a + b + c = 100$ ,
- (a) if  $a, b,$  and  $c$  are all distinct positive integers?
- (b) if  $a, b,$  and  $c$  are all positive integers with  $a < b < c$ ?
- 8 In the  $xy$ - coordinate plane define the distance from the origin  $O$  to a lattice point  $A$  different from  $O$  as the length of the path along the coordinate lines. How many points are at a distance of less than or equal to  $n$  units from  $O$ , where  $n$  is a positive integer?
- 9 The figure below is a  $10 \times 1$  ice cube tray. In how many different ways could you remove 3 of the ice cubes that are not next to each other?



- 10 (a) How many 3-element subsets  $S$  of  $\{1, 2, \dots, 8\}$  have the property that no two consecutive integers belong to  $S$ ?
- (b) How many  $n$ -element subsets  $S$  of  $\{1, 2, 3, \dots, m\}$  have the property that no two consecutive integers belong to  $S$ ?
- (c) How many 4-element subsets  $S$  of  $\{8, 9, \dots, 16\}$  have the property that no two consecutive integers belong to  $S$ ?

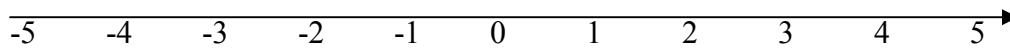
- (d) How many non-empty subsets  $S$  of  $\{1, 2, 3, \dots, 15\}$  have the following two properties?  
 (1) No two consecutive integers belong to  $S$ .  
 (2) If  $S$  contains  $k$  elements, then  $S$  contains no number less than  $k$ .

- 11** A bug moves along the number line from  $O$  with each step either one unit to left or right. In how many different ways that, in ten steps, it will

(a) return to  $O$ ?

(b) reach at  $2$ ?

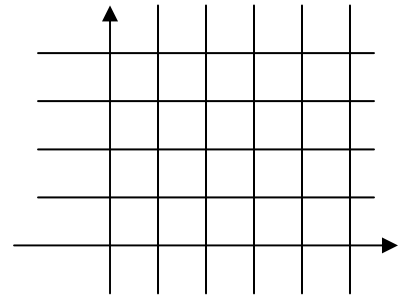
(c) reach at  $-1$ ?



- 12** A bug moves in the coordinate plane from origin  $O(0, 0)$  with each step one unit to the left or right or up or down. In how many different ways that it will end at  $(1, 2)$

(a) after nine moves?

(b) after ten moves?



- 13** (a) How many terms are in the simplified expression of  $(x + y)^{10}$ ?

(b) How many terms are in the simplified expression of  $(x + y + z)^{100}$ ?

(c) How many terms are in the simplified expression of  $(x + y + z)^{2006} + (x - y - z)^{2006}$ ?