Counting and Probability II

Examples:

1 A bug starts at one vertex of a cube and moves along the edges of the cube according to the following rule. At each vertex the bug will choose to travel along one of the three edges emanating from that vertex. Each edge has equal probability of being chosen, and all choices are independent. What is the probability that after seven moves the bug will have visited every vertex exactly once?

(A) $\frac{1}{2187}$ **(B)** $\frac{1}{729}$ **(C)** $\frac{2}{243}$ **(D)** $\frac{1}{81}$ **(E)** $\frac{5}{243}$

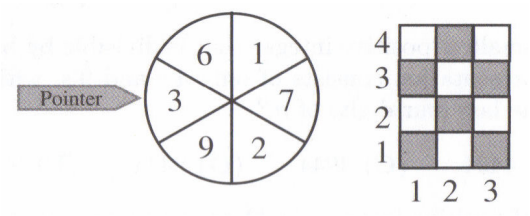
2 For a particular peculiar pair of dice, the probabilities of rolling 1, 2, 3, 4, 5, and 6 on each die are in the ratio 1 : 2 : 3 : 4 : 5 : 6. What is the probability of rolling a total of 7 on the two dice?

(A) $\frac{4}{63}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{8}{63}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{2}{7}$

3 Integer a, b, c, and d, not necessarily distinct, are chosen independently and at random from 0 to 2007, inclusive. What is the probability that ad − bc is even?

(A) $\frac{3}{8}$ (B) $\frac{7}{16}$ (C) $\frac{1}{2}$ (D) $\frac{9}{16}$ (E) $\frac{5}{8}$

4 The wheel shown is spun twice, and the randomly determined numbers opposite the pointer are recorded. The first number is divided by 4, and the second number is divided by 5. The first remainder designates a column, and the second remainder designates a row on the checkerboard shown. What is the probability that the pair of numbers designates a shaded square?



(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

5 A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. What is the expected return to the player, in dollars, for one row of the dice?

(A) $-\frac{1}{8}$ **(B)** $-\frac{1}{16}$ **(C)** 0 **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{8}$

Exercises:

1 An envelope contains eight bills: 2 ones, 2 fives, 2 tens, and 2 twenties. Two bills are drawn at random without replacement. What is the probability that their sum is \$20 or more?

(A) $\frac{1}{4}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

2 Forty slips are place into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let p be the probability that all four slips bear the same number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value q/p?

(A) 162 **(B)** 180 **(C)** 324 **(D)** 360 **(E)** 720

3 A player pays \$5 to play a game. A die is rolled. If the number on the die is odd, the game is lost. If the number in the die is even, the die is rolled again. In this case the player wins if the second number matches the first and loses otherwise. How much should the player win if the game is fair? (In a fair game the probability of winning time the amount won is what the player should pay.)

(A) \$12 **(B)** \$30 **(C)** \$50 **(D)** \$60 **(E)** \$100

4 Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?

(A) $\frac{1}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{4}{5}$ **(E)** 1

5 A set of 25 square blocks is arranged into a 5×5 square. How many different combinations of 3 blocks can be selected from that set so that no two are in the same row or column?

(A) 100 **(B)** 125 **(C)** 600 **(D)** 2300 **(E)** 3600

Homework:

1 All of David's phone numbers have the form 555-*abc*-*defg*, where *a*, *b*, *c*, *d*, *e*, *f*, and *g* are distinct digits and in increasing order, and none is either 0 or 1. How many different telephone numbers can David have?

(A) 1 **(B)** 2 **(C)** 7 **(D)** 8 **(E)** 9

2 A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

(A) $10^4 \cdot 26^2$ **(B)** $10^3 \cdot 26^3$ **(C)** $5 \cdot 10^4 \cdot 26^2$ **(D)** $10^2 \cdot 26^4$ **(E)** $5 \cdot 10^3 \cdot 26^3$

3 How many four-digit positive integers have at least one digit that is a 2 or 3?

(A) 2439 **(B)** 4096 **(C)** 4903 **(D)** 4904 **(E)** 5416

4 Bob and Alice each have a bag that contains one ball of each of the colors blue, green, orange, red, and violet. Alice randomly selects one ball from her bag and puts it into Bob's bag. Bob then randomly selects one ball from his bag and puts it into Alice's bag. What is the probability that after this process the contents of the two bags are the same?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

5 Two tour guides arte leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different grouping of guides and tourists are possible?

(A) 56 **(B)** 58 (C) 60 **(D)** 62 **(E)** 64