

Lectures on Challenging Mathematics

Core Computational Mathematics Volume 3.3

UC3 Geometry

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1.14 Revisiting arcs and angles

1. Regular heptagon $HEXAGON$ is inscribed in a circle. What are the angles in the triangle AGE ?

Let P_1, P_2, \dots, P_n are evenly distributed around a circle. Determine the minimum value of n , given that there are three points P_i, P_j, P_k such that in triangle $P_iP_jP_k$

$$(a) \angle P_i = \frac{180^\circ}{7}, \angle P_j = \frac{360^\circ}{7}, \angle P_k = \frac{720^\circ}{7} \quad (b) \angle P_i = 40^\circ, \angle P_j = 60^\circ, \angle P_k = 80^\circ$$

2. Polygon $A_1A_2 \dots A_n$ is a regular n -gon. For some integer $k < n$, quadrilateral $A_1A_2A_kA_{k+1}$ is a rectangle of area 6. If the area of $A_1A_2 \dots A_n$ is 60, compute n .
3. In triangle ABC , we have $AB = 7$, $AC = 8$, and $BC = 9$. Point D lies on the circumscribed circle of the triangle so that ray AD bisects $\angle BAC$. What is the value of AD/CD ?
4. Let M and A be two given points on circle ω with minor arc $\widehat{MA} = 80^\circ$. Let T and H be two moving points on the major \widehat{MA} with minor arc $\widehat{TH} = 100^\circ$. Chords AH and MT meet at P . As T and H moving along the arc, what is the locus of P ?
5. Distinct points A and B are on a semicircle with diameter MN and center C . Point P lies on segment CN and $\angle CAP = \angle CBP = \alpha$ and $\angle ACM = \beta$. Express $\angle BPN$ in terms of α and β .