Lectures on Challenging Mathematics

Core Computational Mathematics
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UC3 Counting

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1.13 Practices with binomial coefficients (part 1)

1. The PEA mathematics department is to hold a meeting to discuss pedagogy. After a long conversation among 23 members of the department, they decide to split into 5 groups of three and 2 groups of four to continue their discussion. In how many ways can this be done?

2. In the expansion of $(ax+b)^{2000}$, where a and b are relatively prime positive integers, the coefficients of x^2 and x^3 are equal. Find a and b.

3. What is the value of the constant term in the expansion of $\left(\left(x+\frac{1}{x}\right)^2-4\right)^{20}$?

4. There are 10 people who want to choose a committee of 5 people among them. They do this by first electing a set of 1, 2, 3, or 4 committee leaders, who then choose among the remaining people to complete the 5-person committee. In how many ways can the committee be formed, assuming that people are distinguishable? (Two committees that have the same members but different sets of leaders are considered to be distinct.)

5. How many ways are there to place two `A`'s, two `B`'s, two `C`'s, and two `D`'s in four distinguishable boxes such that every box has two letters?