

Lectures on Challenging Mathematics

Core Computational Mathematics Volume 2.3

UC2 Geometry

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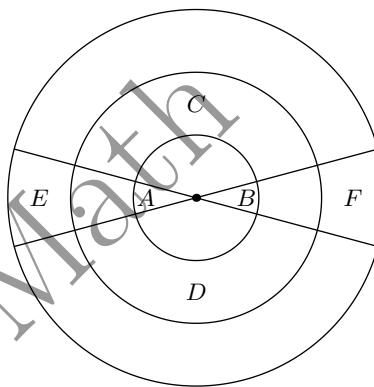
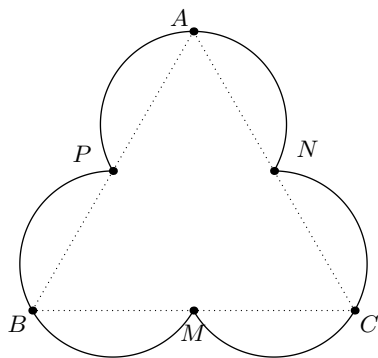
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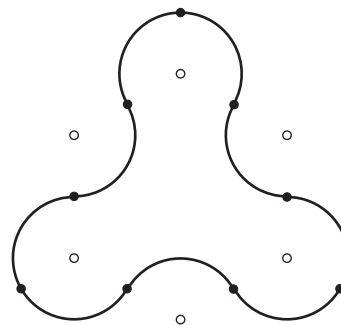
1.15 Circles (part 2)

- Let M, N, P be midpoints of the sides BC, CA, AB of an equilateral triangle ABC with side length 4. Construct parts of the circumcircles of triangles $ANP, BMP,$ and CMN that lie outside of triangle ABC to get a *trefoil* shown below in the left-hand side figure. What is the area of the trefoil?



- Two distinct lines pass through the center of three concentric circles of radii 1, 2, and 3. In the right-hand side figure shown above, the total area of the closed regions labeled A through F is $8/13$ of the total area of unlabeled closed regions. What is the degree measure of the acute angle formed by the two lines?

- The closed curve in the figure is made up of 9 congruent circular arcs each of length $\frac{2\pi}{3}$, where each of the centers of the corresponding circles is among the vertices of a regular hexagon of side 2. What is the area enclosed by the curve?



- Three semicircles of radius 1 are constructed on diameter AB of a semicircle of radius 2. The centers of the small semicircles divide segment AB into four line segments of equal lengths. What is the area of the region that lies within the larger semicircle but outside the smaller semicircles?

- A circular sector made of paper is formed by radii OA and OB and arc \widehat{AB} . This piece of paper is folded along a line passing through B such that point O lands on \widehat{AB} . Given that $\angle AOB = 90^\circ$ and $OA = 1$, find the length of the crease.