

Lectures on Challenging Mathematics

Core Computational Mathematics Volume 2.2

UC2 Counting

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Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

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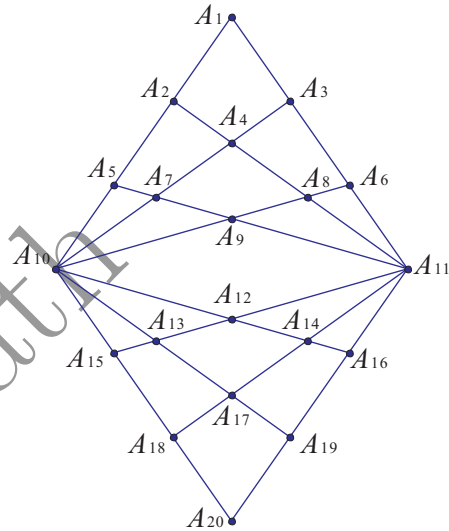
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Internal Use

1.6 Counting practice (part 4)

1. In how many ways can Mr. Fat travel from A_1 to A_{20} along the paths in the figure shown on the right-hand side, assuming that he needs to go downward all of the time?
2. For a permutation $p = (a_1, a_2, \dots, a_9)$ of $(1, 2, \dots, 9)$, let $n(p)$ denote the maximum of the three products $a_1a_2a_3, a_4a_5a_6, a_7a_8a_9$, and let m denote the minimum value of $n(p)$ for all possible permutations p . Determine the number of permutations p with $n(p) = m$.
3. Alex, Mel, and Chelsea play a game that has 6 rounds. In each round there is a single winner, and the outcomes of the rounds are independent. For each round the probability that Alex wins is $\frac{1}{2}$, and Mel is twice as likely to win as Chelsea. What is the probability that Alex wins three rounds, Mel wins two rounds, and Chelsea wins one round?
4. A hat contains four cards, three of which are black on one side and red on the other, while the fourth is red on both sides. Nick chooses a card at random, looks at only one side, and observes that it is red. What is the probability that he has chosen the card that is red on both sides?
5. Determine the number of permutations of the 15-character string **AAAABBBBBBCCCCC** such that
 - (a) None of the first four letters is an **A**;
 - (b) None of the next five letters is a **B**;
 - (c) None of the last six letters is a **C**.



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