Lectures on Challenging Mathematics

Core Computational Mathematics
Volume 2.1

UC2 Algebra

Winter 2018

Zuming Feng
Phillips Exeter Academy and IDEA Math
zfeng@exeter.edu

Contents

1.16 Operations rules with logarithms (part 5)

1. Without calculator, solve for x:
   (a) $\log_4 x = -1.5$ (b) $\log_x 8 = 6$
   (c) $27 = 8(x-2)^3$ (d) $3\log_{27}(x-2)^4 = 2$

2. Rewrite the equation $19\log x - 9 + 9\log y = 199\log 8z$ so that it makes no reference to logarithms.

3. When $10^{3.43429448}$ is evaluated, how many digits are found to the left of the decimal point? You can answer this question without using your calculator. Use your calculator to find its first three digits.

   What are the first three digits when each of the following numbers is evaluated?
   (a) $10^{7.43429448}$ (b) $10^{-0.43429448}$ (c) $10^{0.56570552}$ (d) $10^{-7.4342944}$

4. Earthquakes can be classified by the amount of energy they release. Because of the large numbers involved, this is usually done logarithmically. The Richter scale is defined by the equation $R = 0.67\log(E) - 1.17$, where R is the rating and E is the energy carried by the seismic wave, measured in kilowatt-hours. (A kilowatt-hour is the energy consumed by ten 100-watt light bulbs in an hour.)
   (a) The 1989 earthquake in San Francisco was rated at 7.1. What amount of energy did this earthquake release? It could have sustained how many 100-watt light bulbs for a year?
   (b) An earthquake rated at 8.1 releases more energy than an earthquake rated at 7.1. How many times more?
   (c) Rewrite the defining equation so that E is expressed as a function of R.
   (d) Adding 1 to any rating corresponds to multiplying the energy by what constant?
   (e) Is it possible for a seismic wave to have a *negative* rating? What would that signify?

5. Solve each of the following equations for x.
   (a) $2\log(2x)^3 = 3\log(x-15)^2$ (b) $\log_2 x + \log_x 2 = \dfrac{34}{15}$