Lectures on Challenging Mathematics

Math Challenges 5

Geometry

Summer 2018

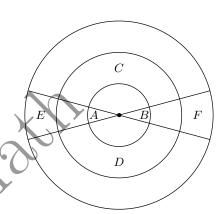
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1.4 Practices with geometric computations (part 2)

- 1. Farmer Chong Gu glues together four equilateral triangles of side length 1 such that their edges coincide. He then drives in a stake at each vertex of the original triangles and puts a rubber band around all the stakes. Find the minimum possible length of the rubber band.
- 2. Two distinct lines pass through the center of three concentric circles of radii 1, 2, and 3. In the right-hand side figure shown above, the total area of the closed regions labeled A through F is 8/13 of the total area of unlabeled closed regions. What is the degree measure of the acute angle formed by the two lines?
 - In rectangle ABCD, we have AB = 8, BC = 9. Point H and E lie on sides BC and AD, respectively, with BH = 6 and DE = 4. Line EC intersects line AH at G, and F is the foot of the perpendicular from G to line AD. Find GF.



- | 4. The incircle of triangle ABC touches the sides AB, BC, CA at F, D, E, respectively. Set AB = c, BC = a, CA = b. Express the length of each of the segments BD, DC, CE, EA, AF, FB in terms of a, b, c.
 - Equilateral triangle XYZ is inscribed in a unit circle ω . Let W be a point other than X in the plane such that triangle WYZ is also equilateral. Determine the area of the region inside triangle WYZ that lies outside circle ω .