Lectures on Challenging Mathematics

Olympiad Math 2

Geometry

Winter 2018

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Contents

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1.13 Angle chasing and cyclic quadrilaterals (part 3)

1. [New Problems in Euclidean Geometry, by David Monk] Let $ABCD$ be a cyclic quadrilateral. The perpendicular bisector of segment CD meets lines AD and BD at P and Q, respectively. Prove that $\angle ACQ = \angle PCB$.

2. Let $ABCD$ be a cyclic quadrilateral with $BC = BA + CD$. Prove that the bisectors of angles A and D intersect on segment BC.

3. [New Problems in Euclidean Geometry, by David Monk] Quadrilateral $ABCD$ is inscribed in circle ω. Diagonals AC and BD meet at R, and rays AD and BC meet at P. Let X, Y, Z be the feet of the perpendiculars from D to lines AC, BC, PR, respectively. Prove that the circumcircle of XYZ passes through the midpoint of segment CD.

4. [New Problems in Euclidean Geometry, by David Monk] In triangle ABC, $AB \neq AC$. Point D is the foot of the perpendicular from A to line BC. Points M and N are the midpoints of segments BC and AD, respectively. Point P is the foot of the perpendicular from B to line AM. Prove that line MN is tangent to the circumcircle of CMP.

5. Let ABC be an acute triangle, and let AA_1, BB_1, and CC_1 be its altitudes. Segments AA_1 and B_1C_1 meet at point K. The perpendicular bisector of segment A_1K intersects sides AB and AC at L and M, respectively. Prove that points A, A_1, L, and M lie on a circle.

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