

Lectures on Challenging Mathematics

Olympiad Math 1

Geometry

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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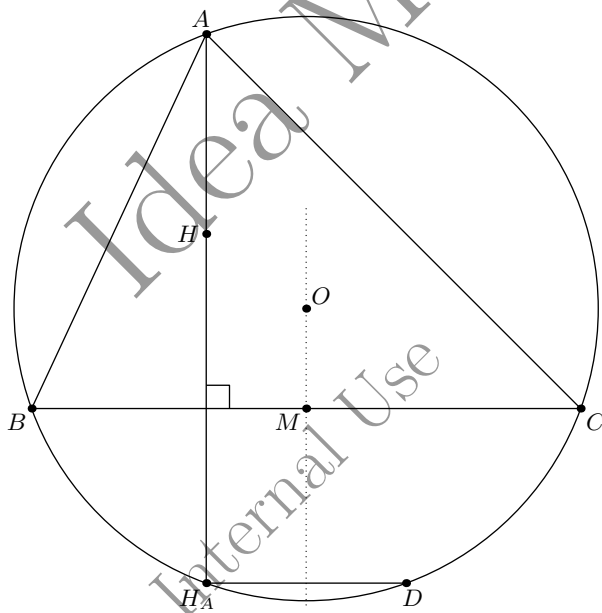
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1.10 Angle chasing and the centers of triangles (part 2)

1. The altitudes BE and CF in triangle ABC intersect in point H . Denote by O the circumcenter of triangle ABC . Prove that line AO is perpendicular to line EF .
2. The altitudes AA_1 , BB_1 , CC_1 of an acute triangle ABC intersect in point H . Show that the orthocenter of triangle ABC , H , is the incenter of triangle $A_1B_1C_1$. What roles do points A, B, C play relative to triangle $A_1B_1C_1$?
3. (Continuation) Suppose ABC is an obtuse triangle with $\angle A > 90^\circ$. Describe the orthocenter of triangle ABC in relation with triangle $A_1B_1C_1$.
4. Let ABC be an acute triangle with circumcircle ω . Let O and H denote its circumcenter and orthocenter. Denote by M be the midpoint of BC . Point H_A lies on \widehat{BC} (not including A) such that $AH_A \perp BC$. Let D be the reflection of H_A across line OM .



Prove that

- (a) H and H_A are symmetric across the line BC .
 - (b) points H , M , and D are collinear.
5. In parallelogram $ABCD$, point H is the orthocenter of triangle ABC . The line through H parallel to line AB meets line BC at P and AD at Q ; the line through H parallel to line BC meets line AB at R and CD at S . Prove that P, Q, R, S lie on a circle.