Lectures on Challenging Mathematics

Math Challenges 3

Number Sense

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1.9 Multiples of small numbers (part 1)

1. Complete the following sentence:

   > If the sum of three integers is divisible by 3, then the remainders of these integers divided by 3 are either ... or ....

2. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37^{th} and 64^{th}, respectively. How many schools are in the city?

3. Consider the statement:

   > Positive integer $n = \overline{a_h a_{h-1} \dots a_1 a_0}$ is divisible by 7 if and only if the difference $m = \overline{a_h a_{h-1} \dots a_1} - 2 \cdot a_0$ is divisible by 7.

   Understand this statement by checking at least 5 numerical examples that no one else in the class will think of. Show that this statement is true.

4. Let m be a positive integer divisible by 99. Show that integer n obtained by reversing the order of the digits of m is also divisible by 99.

5. Find the smallest positive integer n such that the decimal representation of $2010n$ has exactly one even digit.