

# Lectures on Challenging Mathematics

## Integrated Mathematics 2

### Algebra (Part 1)

Winter 2018

Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

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### 1.13 Revisiting rational expressions (part 2)

1. If  $\frac{91x - 117y}{39y - 65y} = \frac{3}{2}$ , what is  $\frac{x}{y}$ ?
2. Explain your opinions of each of the following student responses:
  - (a) Asked to find an expression equivalent to  $x^9 - x^3$ , a student responded  $x^6$ .
  - (b) Asked to find an expression equivalent to  $x^9/x^3$ , a student responded  $x^3$ .
  - (c) Asked to find an expression equivalent to  $\frac{x^9 - x^6}{x^3}$ , a student responded  $x^6 - x^3$  and another student said  $x^3 - x^2$ .
  - (d) One student said that  $\frac{x^3}{x^9 - x^6}$  is equivalent to  $\frac{1}{x^6} - \frac{1}{x^3}$ , another student said it is equivalent to  $\frac{1}{x^3} - \frac{1}{x^2}$ .
3. Even though  $16x^4 - 2401$  is a 4<sup>th</sup> degree polynomial. It can be factored as a quadratic. If you have trouble seeing this, try to set  $u = x^2$  first and then complete the factoring (in  $x$ ).
4. Rewrite each of the following as a single fraction.
  - (a)  $ab^{-1} - 2ba^{-1}$
  - (c)  $\frac{1}{x} - \frac{2}{x+1} + \frac{3}{x+2}$
5. Find all integers  $m$  for which  $\frac{4m+30}{2m-5}$  is an integer.

## 1.18 The second look at quadratics and parabolas (part 2)

1. Devise a quick way to write an equation for the symmetry axis of a parabola of the form  $y = ax^2 + bx$ . What is the equation for the axis of symmetry of a parabola  $y = ax^2 + bx + c$ ?

2. Simplify  $\frac{p^2 - 6pq + 8q^2}{p^2 - 2pq} \div (p^2 - 3pq - 4q^2)$

3. If I were to increase my cycling speed by 3 mph, it would take me 40 seconds less time to cover each mile. What is my current cycling speed?

4. Avery and Sasha were comparing parabola graphs on their calculators. Avery had drawn  $y = 0.001x^2$  in the window  $-1000 \leq x \leq 1000$  and  $0 \leq y \leq 1000$ , and Sasha had drawn  $y = x^2$  in the window  $-k \leq x \leq k$  and  $0 \leq y \leq k$ . Except for scale markings on the axes, the graphs looked exactly the same! What was the value of  $k$ ?

5. Using a driver on the 8<sup>th</sup> tee, which is on a plateau 10 meters above the level fairway, Dale hits another fine shot. The quadratic function  $y = 10 + 0.5x - 0.002x^2$  describes this parabolic trajectory of the shot. Why should you expect this tee shot to go more than 250 meters?

To find the length of the shot, follow these steps:

- Set  $y$  equal to 0 and solve for  $x$ . Explain why, and show how to arrive at  $x^2 - 250x = 5000$ .
- Add  $125^2$  to both sides of this equation. Why was this number chosen?
- Complete the solution and find the length of the shot.
- Comment on the presence of the number 125 in the answer. What is its significance?