

Lectures on Challenging Mathematics

UC3M1

Invitation to Computational Mathematics Part 3, Module 1

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Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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Contents

Algebra	1
1.1 Rational and radical expressions	1
1.2 Vieta’s relation for the quadratic equation (part 1)	3
1.3 Computations with logarithm	4
1.4 Vieta’s relation for the quadratic equation (part 2)	6
1.5 More on distance and motion	7
1.6 The Quadratic function and its graph (part 1)	9
1.6.1 The Quadratic function and its graph	9
1.6.2 Mixed exercises	10
1.7 Visiting the parabola and its tangent lines from the view of conic curves	11
1.8 The quadratic function and its graph (part 2)	12
1.8.1 The quadratic function and its graph	12
1.8.2 Mixed exercises	13
1.9 Sums and products	14
1.10 Algebra 2.1	15
1.10.1 Practice set 1	15
1.10.2 Practice set 2	15
1.13 A short review on the quadratic function (part 2)	21
1.12 Algebra 2.2	19
1.12.1 Practice set 3	19
1.12.2 Practice set 4	19
1.13 A short review on the quadratic function (part 2)	21
1.14 Algebra 2.3	23
1.14.1 Practice set 5	23
1.14.2 Practice set 6	23
1.15 A short review on the quadratic function (part 3)	24
1.16 Algebra 2.4	25
1.16.1 Practice set 7	25
1.16.2 Practice set 8	25
1.17 Algebra 2.5	26
1.17.1 Practice set 9	26
1.17.2 Practice set 10	26

2	Number Theory	27
2.1	Computations with G.C.D. and L.C.M.	27
2.1.1	Computations with G.C.D. and L.C.M.	27
2.1.2	Divisors and their sums	28
2.2	Divisors	29
2.3	Computational practices (part 1)	31
2.3.1	Computational practices (part 1)	31
2.3.2	Establishing the Frobenius Coin theorem (part 1)	31
2.4	Computational practices (part 2)	33
2.4.1	Computational practices (part 2)	33
2.4.2	Establishing the Frobenius Coin theorem (part 2)	34
3	Practice tests	35
3.1	Entry level Combinatorics problems from the first 10 years of AIME	35
3.2	Entry level Geometry problems from the first 10 years of AIME	37
3.3	Entry level Number Theory problems from the first 10 years of AIME	39
3.4	Selected entry level problems from Mandelbrot 2013	41
3.4.1	Practice set 1	41
3.4.2	Practice set 2	41
3.4.3	Practice set 3	42