

Lectures on Challenging Mathematics

RP3M2

Elements of Mathematics Olympiad Part 3, Modulo 2

Summer 2014

Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

©Copyright 2008 – 2014 Idea Math

Copyright © 2008 – 2014 IDEA MATH.

“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

©Copyright 2008 – 2014 Idea Math

Idea Math
Internal Use

Contents

©Copyright 2008-2014 Idea Math

1 Algebra	1
1.1 Techniques in solving inequalities – completing the squares	1
1.2 Techniques in solving inequalities – are you Schur?	3
1.3 Techniques in solving inequalities – revisiting Cauchy-Schwartz	5
1.3.1 A generalized form of Cauchy-Schwartz	5
1.3.2 Revisiting Cauchy-Schwartz	5
1.4 Techniques in solving inequalities – (weighted) AM-GM	7
2 Combinatorics	9
2.1 Proof exercises (part 1)	9
2.2 Proof exercises (part 2)	12
2.3 Proof exercises (part 3)	14
3 Geometry	17
3.1 Law of cosines and Brocard points	17
3.1.1 Law of cosines and Brocard points	17
3.1.2 Pole and polar map (part 1) – definitions and properties	18
3.2 Law of sines and Ceva’s theorem	20
3.2.1 Law of sines and Ceva’s theorem	20
3.2.2 Pole and polar map (part 2) – practices	21
3.3 Menelaus’ theorem in action	22
3.3.1 Menelaus’ theorem in action	22
3.3.2 A light touch on useful geometry facts for non-synthetic approach	23
3.4 Power-of-a-point in proofs	24
3.4.1 Pole and polar map (part 3 – Brocard’s theorem)	25
3.5 Spiral similarity and homothety	26
3.5.1 Spiral similarity	26
3.5.2 Homothety	26
3.5.3 Challenges with centers of triangles (part 1)	27
3.6 Spiral similarity and intersecting circles	28
3.6.1 Spiral similarity and intersecting circles	28
3.6.2 Challenges with centers of triangles (part 2)	29

4	Number Theory	31
4.1	A short review on Diophantine equations and floor functions (part 2)	31
4.2	Modulo arithmetic and Diophantine equations	33
4.2.1	Modulo	33
4.2.2	Pythagorean Equations	33
5	Practice tests	35
5.1	RP3M2 practice test 1	35
5.2	RP3M2 practice test 2	36
5.3	RP3M2 practice test 3	37
5.4	RP3M2 practice test 4	38
5.5	RP3M2 practice test 5	39