

Lectures on Challenging Mathematics

RC2M2

Enhancement of Computational Mathematics Part 1, Modulo 2

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Zuming Feng

Phillips Exeter Academy and IDEA Math
zfeng@exeter.edu

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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