Lectures on Challenging Mathematics

RC1M2
A Path from (advanced) Pre-Calculus to (higher) contests mathematics Modulo 2

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"Cogito ergo Sum" – "I think, therefore I am"

René Descartes (1596-1650)

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