

# Lectures on Challenging Mathematics

## PCXM2

### Essential Computational Mathematics Part 3, Modulo 2

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*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596-1650)

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