

Lectures on Challenging Mathematics

EP1M1

Challenges in Mathematics Olympiad Part 1, Modulo 1

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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