1.4 Cubes and their sums and differences (part 1)

1. Mentally find the prime factorizations of $9^3 - 4^3$ and $25^3 - 27^2$.

2. Given that $a = \sqrt[3]{5} + 7$, explain why a is a root of the equation $(x-7)^3 - 5 = 0$. Expand this polynomial.

3. Factor $(a+b)^3 - (a^3 + b^3)$ and $(a-b)^3 - (a^3 - b^3)$. Then solve the equation
$$\sqrt[3]{7x+19} - \sqrt[3]{7x-19} = \sqrt[3]{2}.$$

4. Find the 4-digit prime that divides 1000001. Factor 999999.

5. Determine the largest prime factor of $3^{15} + 3^{11} + 3^6 + 1$.

1.9 Playing with numbers (part 2)

1. In the *2, 3, 5, 7 game*, players count the positive integers, starting with 1 and increasing, which do not contain the digits 2, 3, 5, and 7, and also are not divisible by the numbers 2, 3, 5, and 7. What is the seventh number counted?

2. Find the smallest positive integer n such that $1^2 + 2^2 + \cdots + n^2$ is divisible by 325.

3. Given that m and n are positive integers such that $6^m \cdot 8^n$ divides 2014!, find the maximum value of $m + n$.

4. Suppose that p and q are two-digit prime numbers such that $p^2 - q^2 = 2p + 6q + 8$. Compute the largest possible value of $p + q$.

5. Consider n concentric circles with radii $1, 2, \ldots, n$. The total area that is contained inside an *odd* number of these circles is $m\pi$ for a positive integer m. Find the least n such that m is a multiple of 1001.