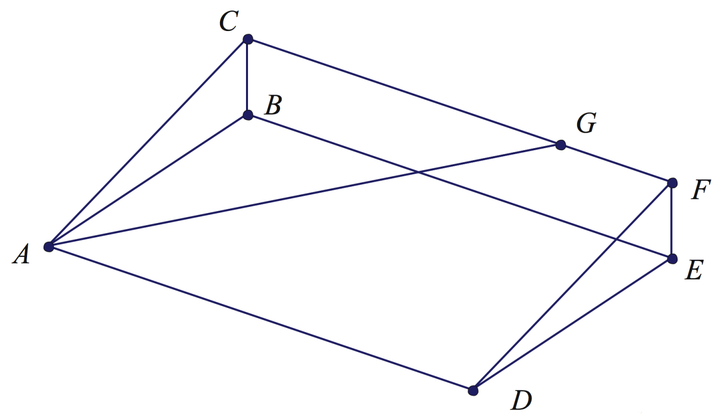
1.19 Pythagorean theorem (part 3)

1. Two angles of a triangle measure 30 and 45 degrees. If the side of the triangle opposite the 30-degree angle measures 12 units, what is the sum of the lengths of the two remaining sides?

2. In triangle ABC, $\angle C = 90°$. Medians are drawn from point A and point B in this right triangle to divide segment BC and AC in half, respectively. The lengths of the medians are 6 and $2\sqrt{11}$ units, respectively. How many units are in the length of segment AB?

3. Let DEF be a triangle and H the foot of the altitude from D to EF. If $DE = 60$, $DF = 35$, and $DH = 21$, what is the difference between the minimum and the maximum possible values for the area of DEF?

4. Peyton's workout today is to run repeatedly up a steep grassy slope, represented by $ADFC$ in the diagram. The workout loop is $AGCA$, in which AG requires exertion and GCA is for recovery. Point G was chosen on the ridge CF to make the slope of the climb equal 20%.



Given that $ADEB$ and $BEFC$ are rectangles, ABC is a right angle, $AD = 240$, $DE = 150$, and $EF = 50$, find the distance from point G to point C.

5. (Continuation) Peyton's next workout loop is $AHCA$, where H is a point on the path AG, chosen to make the slope of HC equal 20%. Find the ratio AH/AG, and explain your choice.

1.25 Similarity of polygons

1. Two polygons are *similar* if corresponding sides taken in the same sequence are proportional and corresponding angles taken in the same sequence are equal in measure.

   (a) The AAA rule guarantees that two triangles are similar. Does the $AAAA$ rule guarantee that two quadrilaterals are similar?

   (b) The SSS rule guarantees that two triangles are similar. Does the $SSSS$ rule guarantee that two quadrilaterals are similar?

2. A rectangular sheet of paper is 21 cm wide. When it is folded in half, with the crease running parallel to the 21-cm sides, the resulting rectangle has the same shape as the unfolded sheet. Find the length of the sheet. Note: in many places outside of the United States, such as Europe, the shape of the notebook paper is determined by this similarity property.

3. Let $ABCDEF$ be a regular hexagon. If the area of the region enclosed by diagonals AC, BD, CE, DF, EA, FB is equal to 2, what is the area $ABCDEF$?

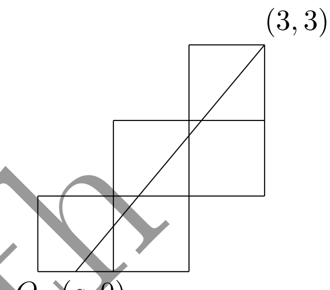
4. In parallelogram $ABCD$, $AB = 2$ and $BC = 3$, and $\angle A = 60°$. Extend segment AB through B to P such that BCP is an equilateral triangle. Extended segment CD through D to Q such that ADQ is an equilateral triangle. Is $APCQ$ a parallelogram? If so, is it similar to $ABCD$?

5. (Continuation) What should be the ratio AB/AC in a parallelogram $ABCD$ so that the construction mentioned above would yield a parallelogram $APCQ$ similar to $ABCD$?

2.9 Practices in geometric computations (part 5)

1. In triangle ABC, $AB = AC$. Points D and E lie on sides AB and AC respectively such that $AD = DE = EB = BC$. Find $\angle A$.

2. Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin O. The slanted line, extended from $(a, 0)$ to $(3, 3)$, divides the entire region into two regions of equal area. What is a?



3. Let $ABCD$ be a trapezoid with $AB \parallel CD$. Denote by M and N the midpoints of AB and CD. Suppose that $AD = 4$, $AB = 7$, $BC = 3$, and $CD = 12$. Find the length of the segment MN.

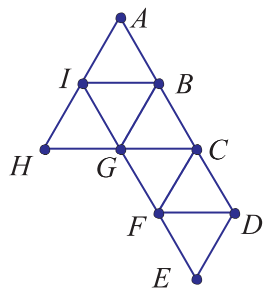
4. Consider a rectangle $ABCD$ with side lengths $AB = CD = 6$ and $BC = DA = 8$. Let X be a point on BD such that $AX \perp BD$. Find the length of each of following segments:

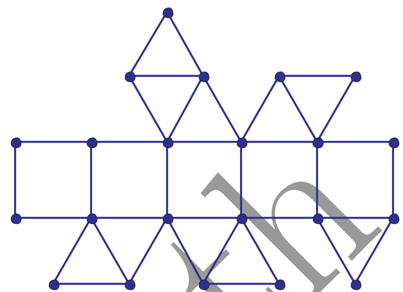
   (a) AX (b) BX (c) CX

5. Let $ABCD$ be a square. Points P and Q are variable points on segments AB and CD respectively such that the ratio between the areas of quadrilaterals $APQD$ and $BPQC$ is equal to $2 : 3$. Show that line PQ passes through a fixed point.

3.3 2-D and 3-D vision (part 2)

1. The grid, in the left-hand side figure shown below, will be folded to form an octahedron, but one face will be missing. Which three edges will form the boundary of the missing triangular face?





2. This net, in the right-hand side figure shown above, with 5 square faces and 10 equilateral triangular faces is folded into a 15-faced polyhedron. How many edges does the polyhedron have?

3. An ant is positioned at A, one of the eight vertices of a solid $4 \times 6 \times 8$ rectangular box. It needs to crawl to vertex B, which is the furthest vertex from A, as fast as possible. Find one of the shortest routes. How many are there?

4. Tile a 5×8 rectangle by using two of each 5 incongruent tetrominoes.

5. (Continuation) Tile a 4×10 rectangle by using two of each 5 incongruent tetrominoes.

3.4 Selected entry level geometry problems from AIME

1. A circle with diameter PQ of length 10 is internally tangent at P to a circle of radius 20. Square $ABCD$ is constructed with A and B on the larger circle, side CD tangent at Q to the smaller circle, and the smaller circle outside $ABCD$. The length of AB can be written in the form $m + \sqrt{n}$, where m and n are integers. Find $m + n$.

2. Let u and v be integers satisfying $0 < v < u$. Let $A = (u, v)$, let B be the reflection of A across the line $y = x$, let C be the reflection of B across the y-axis, let D be the reflection of C across the x-axis, and let E be the reflection of D across the y-axis. The area of the pentagon $ABCDE$ is 451. Find $u + v$.

3. When a right triangle is rotated about one leg, the volume of the cone produced is 800π cm^3. When the triangle is rotated about the other leg, the volume of the cone produced is 1920π cm^3. What is the length (in cm) of the hypotenuse of the triangle?

4. Rectangle $ABCD$ has sides AB of length 4 and CB of length 3. Divide AB into 168 congruent segments with points $A = P_0, P_1, \ldots, P_{168} = B$, and divide CB into 168 congruent segments with points $C = Q_0, Q_1, \ldots, Q_{168} = B$. For $1 \le k \le 167$, draw segment P_kQ_k. Repeat this construction on the sides AD and CD, and then draw the diagonal AC. Find the sum of the lengths of the 335 parallel segments.

5. A wooden cube, whose edges are one centimeter long, rests on a horizontal surface. Illuminated by a point source of light that is x centimeters directly above an upper vertex, the cube casts a shadow on the horizontal surface. The area of the shadow, which does not include the area beneath the cube, is 48 square centimeters. Find the greatest integer that does not exceed $1000x$.