1.5 Simple counting practices (part 5)

1. On a regular ruler (say, of length 10, with 11 marks, namely, 0, 1, 2, . . . , 10), seven marks are to be erased. If we want to be able to measure all integer distances up to n. What is the maximum number of n?

2. Six people, all of different weights, are trying to build a human pyramid: that is, they get into the formation shown on the right. We say that someone not in the bottom row is *supported by* each of the two closest people beneath her or him. How many different pyramids are possible, if nobody can be supported by anybody of lower weight?

   ```
     A
    B C
   D E F
   ```

3. Alex is given a number. Every second, Alex can add or subtract any number of the form $n!$ to his current number to get a new number. In how many ways can Alex get from 0 to 100 in 4 seconds?

4. Using the letters `S, I, M, P, L, E` we form distinct six-letter codes. If these codes are arranged in alphabetical order, then what position does the code `SIMPLE` occupy?

5. Determine the number of positive integer divisors of the product $3! \cdot 5! \cdot 7! \cdot 9! \cdot 11!$ that are perfect squares.

6. At Wash College of Higher Education (Wash Ed.), the entering class has n students. Each day, two of these students are selected to oil the slide rules. If the entering class had two more students, there would be 75 more ways of selecting the two slide rule oilers. Compute n.

7. The product of all the positive integer factors of 81^{36} can be written in the form of a^b, what is the minimum value of $a + b$ if a and b are positive integers?

8. How many positive integers between 1 and 125, inclusive, are divisible by 4 or 5 but not by 6?

9. The four faces of a fair tetrahedron die are numbered 1, 2, 3, 4. Each time the die is rolled, three numbers are visible. If the die is rolled five times, the sum, denoted by s, of all fifteen of the numbers showing is calculated. What is the probability that $s = 36$? What is the probability that $s = 35$?

10. Suppose that 5-letter code are formed using only the letters `A, R, M, L`. Each letter need not be used in a word, but each word must contain at least two distinct letters. Compute the number of such words that use the letter `A` more than any other letter.

1.9 Counting practices (part 4)

1. Mary insists on sitting next to Bob and Bob insists on sitting next to Jane. Compute the number of different ways that Bob, Mary, Jane, and 3 other students can sit in a row.

2. How many subsets of the set {`M, A, T, H, C, O, U, R, S, E`} contain at least one vowel?

3. Bill is sent to a donut shop to purchase exactly six donuts. If the shop has four kinds of donuts and Bill is to get at least one of each kind, how many combinations will satisfy Bill's order requirements?

4. How many subsets of the set $\{1, 2, 3, \ldots, 30\}$ have the property that the sum of their elements is greater than 232?

5. Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that such three-letter three-digit arrangement is equally likely, the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left) is m/n, where m and n are relatively prime positive integers. Find $m + n$.

6. A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not by 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

7. Zeb has twenty $1 \times 2 \times \pi$ bricks. He stacks them, one on top of another, to form a tower twenty bricks high. Each brick can be in any orientation so long as it rests at on top of the next brick below it (or on the floor). How many distinct heights of towers can he make?

8. Some students in a gym class are wearing blue jerseys, and the rest are wearing red jerseys. There are exactly 25 ways to pick a team of three players that includes at least one player wearing each color. Compute the number of students in the class.

9. How many distinct non-regular polygons can be formed by connecting some different vertices of a regular dodecagon?

10. How many quadruples (a, b, c, d) of 1-digit positive integers are there which satisfies $a + bcd = ab + cd$?