Chapter 1

Algebra Knowledge

1.1 Deriving Quadratic Formula (part 1)

1. Algebra teacher Mr. Fat asked his students Divid, Sqart, Goodstart, and Korrect to solve the *quadratic equation* $x^2 - 4x - 7 = 0$.

   Divid's answer was $x = 4 + \dfrac{7}{x}$. Sqart's answer was $x = \sqrt{4x + 7}$. Comment on these answers.

   Goodstart reformulated the equation as the following.

   (a) $x^2 - 4x - 7 = 0$ (b) $x^2 - 4x + 4 = 7 + 4$ (c) $(x - 2)^2 = 11$

   Korrect claimed that Goodstart's work can be developed to a correct solution, How?

2. Find integers a, b, c, d, e, f, g (where $e > 1$) such that the following set of equations are equivalent to each other.

   (a) $x^2 + 22x + 101 = 0$ (b) $x^2 + 22x = a$
   (c) $x^2 + 22x + b = -101 + b$ (d) $(x + c)^2 = -101 + b$
   (e) $x + c = \pm\sqrt{d}$ (f) $x = e \pm f\sqrt{g}$

3. Find numbers (not necessarily integers) a, b, c, d, e such that the following set of equations are equivalent to each other.

   (a) $x^2 - 19x + 20 = 0$ (b) $x^2 - 19x = a$
   (c) $(x + b)^2 = -20 + c$ (d) $x = d \pm \sqrt{e}$

4. Solve the following equations.

   (a) $x^2 - 6x = 9$ (b) $x^2 - 10x + 15 = 0$ (c) $x^2 - 11x - 16 = 0$

5. Solve the equation $x^2 + bx + c = 0$ for x in terms of b and c.

1.10 Practice with exponents (part 1)

1. Given that $\sqrt[5]{x} = \sqrt[9]{y} = \sqrt[10]{z} = 8$ and $z^a = (xy)^b$ for positive integers a and b. Find the minimal value of $a + b$.

2. Let a and b be two (not necessarily distinct) numbers in the set $\{2, 3, 4, 5\}$, arrange all the numbers in form of $a\sqrt{b}$ in increasing order from left to right. (It might be helpful to convert all the numbers in form of $\sqrt{c}$.)

3. Evaluate $\sqrt{\dfrac{4^9 + 4^9 + 4^9 + 4^9 + 8^{10}}{8^4 + 4^{11}}}$.

4. How many positive integers n satisfy the following condition:
$$(130n)^{50} > n^{100} > 2^{200}?$$

5. Solve the equation $2^{2^n} = 4^{4^{4^4}}$ for n.