1.9 Linear parametric equations (part 5)

1. From its initial position at $(12, 7)$, a bug crawls linearly with constant speed and direction. It passes $(3, 3)$ after two seconds.

   (a) What is the position of the bug after t seconds?

   (b) What is the total *distance* the bug will crawl in the *first* quadrant?

   (c) How much *time* does the bug spend in the *second* quadrant?

2. Bug Aft starts at $A = (7, 2)$ and crawls with a constant speed toward to $B = (-8, 10)$. Aft started at 12:00 pm and finished his trip at 1:00 pm.

   (a) What is the displacement vector (denoted by $\mathbf{u}$) of Aft?

   (b) Find a vector of length 15 in the same direction of $\mathbf{u}$.

   (c) Find a vector of length a in the opposite direction of $\mathbf{u}$.

   (d) Determine the position of Aft at 12:35 PM.

3. On the same coordinate-axis system, graph the line defined by $P_t = (3t - 4, 2t - 1)$ and the line defined by $4x + 3y = 18$. The graphs should intersect in the first quadrant.

   (a) Calculate P_2, and show that it is not the point of intersection.

   (b) Find the value of t for which P_t is on the line $4x + 3y = 18$.

4. A car traveling east at 45 miles per hour passes a certain intersection at 3 pm. Another car traveling north at 60 miles per hour passes the same intersection 25 minutes later. To the nearest minute, figure out when the cars are exactly 40 miles apart.

5. In triangle ABC, $A = (0, 0)$, $B = (4, 3)$, $C = (5, 12)$. Point D lies on side BC such that $\angle BAD = \angle CAD$. Find the coordinates of D.

1.16 Vector motion (part 5)

1. Let $A = (3, 2)$, $B = (1, 5)$, and $P = (x, y)$.
   (a) Find x- and y-values that make ABP a right angle.
   (b) Describe the configuration of all such points P.
2. Let $A = (5, -3, 6)$, $B = (0, 0, 0)$, and $C = (3, 7, 1)$. Show that
   (a) $\angle ABC$ is a right angle.
   (b) the vectors $[5, -3, 6]$ and $[3, 7, 1]$ are perpendicular.
3. Write an equation that says that
   (a) points $(0, 0, 0)$, (a, b, c), and (m, n, p) form a right triangle, the right angle being at the origin. Simplify your equation as much as you can.
   (b) vectors $[a, b, c]$ and $[m, n, p]$ are perpendicular.
   (c) vectors $[a, b]$ and $[m, n]$ are perpendicular.
4. Given vector $\mathbf{u} = [21, -20]$, find a vector
   (a) of length 15 in the same direction of $\mathbf{u}$.
   (b) $[6, k]$ that is perpendicular to $\mathbf{u}$.
   (c) of length a in the opposite direction of $\mathbf{u}$.
5. Given points $A = (3, -8, 3)$ and $B = (7, 4, -4)$. Find a point P on the y-axis such that $\angle APB = 90°$.

1.27 3-D rectangular coordinates and linear equation (part 2)

1. The edges of rectangular box $ABCDEFGH$ are parallel to the coordinate axes, and two of its corners are $A = (2, 1, 3)$ and $G = (9, 5, 7)$, two of its edges are AE and BF, and two of its faces are $ABCD$ and $EFGH$.

   (a) Find coordinates for the other six vertices;

   (b) Find the lengths AH, AC, AF, FD, and AG;

   (c) Find the distance from G to the xy-plane;

   (d) Find the distance from G to the z-axis;

   (e) Find what C, D, H, and G have in common.

   (f) Is angle FCH a right angle? Explain.

   (g) Find the areas of quadrilaterals $CDEF$ and $CAEG$.

   (h) Show that every pair of interior diagonals (such as FD and CE) bisect each other. (You can do this by either a coordinate geometry method or a synthetic geometry method. Try both approaches.)

   (i) Determine if any pairs of interior diagonals intersect each other perpendicularly. (You can do this by either a coordinate geometry method or a synthetic geometry method. Try both approaches.)

2. An airplane that took off from its airport at noon ($t = 0$ hours) moved according to the formula $(x, y, z) = (15, -20, 0) + t[450, -600, 20]$. What is the meaning of the coordinate 0 in the equation? After twelve minutes, the airplane flew over Bethlehem. Where is the airport in relation to Bethlehem, and how high (in km) was the airplane above the town? What (unrealistic) assumptions did you make in answering this question?

3. Cory is swimming in the pool, according to the equation $C_t = (23 - 2t, 47 - 6t, 92 - 9t)$. Katrina is also swimming in the pool, according to the equation $K_t = (31-2t, 32-3t, 89-6t)$. Determine if the paths of Cory and Katrina intersect or not, and if so, will they collide?

4. (Continuation) Mia and Tiffany are sitting at $P = (1, 25, 4)$ having a chat. Find the position of Cory when the distance between her and Mia and Tiffany is minimum.

5. In a coordinate space, determine if there exists a regular tetrahedron with all of its vertices being lattice points.

1.30 Mixed exercises (part 5)

1. While the Wood's Hole-Martha's Vineyard ferryboat steamed along at 8 mph through calm seas, passenger Dale exercised by walking the perimeter of the rectangular deck, at a steady 4 mph. Discuss the variations in Dale's speed *relative to the water*.

2. Given $A = (2, 3, 5)$ and $B = (7, 11, 13)$, find coordinates for points C and D that make $ABCD$ a rectangle such that $\angle ABD = 60^\circ$. Is the answer unique? If yes, explain; if no, how many answers are there?

3. Let $ABCD$ and $EFGH$ be two faces of rectangular box $ABCDEFGH$, with AE and BF being edges of the box. Suppose that $A = (0, 0, 0)$, $G = (4, 3, 2)$, and the sides parallel to the coordinate axes. The midpoint of FG is M.
   (a) Find coordinates for M.
   (b) Find coordinates for the point P on segment AC that is 2 units from A.
   (c) Decide whether angle APM is a right angle, and give your reasons.
   (d) Find the point on segment AC that is closest to M.

4. Given square $ABCD$, choose a point O that is not outside the square and form the vector $\mathbf{v} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$. By trying various possible positions for O, find the shortest and longest possible $\mathbf{v}$.

5. A car travels due east at c mile per minute on a long, straight road. At the same time, a circular storm, whose radius is 56 miles, moves southeast at $\frac{\sqrt{2}}{2}$ mile per minute. At time $t = 0$, the center of the storm is 106 miles due north of the car. Find all possible values of c such that the car never enters the storm circle.