4.3 Computation with floor and ceiling functions (part 1)

1. Solve the following system of equations:

$$\begin{aligned} x + \lfloor y \rfloor + \{z\} &= 200.0, \\ \{x\} + y + \lfloor z \rfloor &= 190.1, \\ \lfloor x \rfloor + \{y\} + z &= 178.8. \end{aligned}$$

2. In the sixth, seventh, eighth, and ninth basketball games of the season, a player scored 23, 14, 11, and 20 points, respectively. Her points-per-game average was higher after nine games than it was after the first five games. If her average after ten games was greater than 18, what is the least number of points she could have scored in the tenth game?

3. For $1 \le x \le 10080$, compute the least upper bound of the solution set to the equation $\lfloor \log_{10} x \rfloor = \log_{10} \lfloor x \rfloor$.

4. Let x be chosen at random from the interval $(0, 1)$. What is the probability that

$$\lfloor \log_{10} 4x \rfloor - \lfloor \log_{10} x \rfloor = 0?$$

5. Compute the area of the solution set of $\lfloor x \rfloor \cdot \lfloor y \rfloor = 2000$.

8.5 Advanced number sense (part 4)

1. Numbers in the set $S = \{1, 2, \ldots, 15\}$ are divided into two nonempty sets S_1 and S_2 such that the product p_1 of the elements in set S_1 is divisible by the product p_2 of the elements in set S_2. Let m denote the minimum value of p_1/p_2. Find the remainder when m is divided by 1000.

2. Given that $n = 3200021$ is the product of three primes, compute the sum of the three primes. What if $n = 3360001$?

3. Let k be the least common multiple of the numbers in the set $\mathcal{S} = \{1, 2, \ldots, 30\}$. Determine the number of positive integer divisors of k that are divisible by exactly 28 of the numbers in the set $\mathcal{S}$.

4. Given a positive integer n, let $p(n)$ be the product of the nonzero digits of n. (If n has only one digit, then $p(n)$ is equal to that digit.) Let

$$S = p(1) + p(2) + \cdots + p(999).$$

   What is the largest prime factor of S?

5. For a positive integer n, let $\sigma(n)$ denote the sum of all the divisors of n. Prove that

$$\sigma(1) + \sigma(2) + \cdots + \sigma(n) \le n^2.$$