1.2 Absolute value functions

1. What is the area of the region defined by the inequality $|3x - 18| + |2y + 7| \leq 3$?

2. Ashley, Betty, Carlos, Dick, and Elgin went shopping. Each had a whole number of dollars to spend, and together they had \$56. The absolute difference between the amounts Ashley and Betty had to spend was \$19. The absolute difference between the amounts Betty and Carlos had was \$7, between Carlos and Dick was \$5, between Dick and Elgin was \$4, and between Elgin and Ashley was \$11. How much did Elgin have?

3. Let $f(x)$ have the form $f(x) = |ax + b| + |cx + d| + e$, where a, b, c, d and e are real numbers. The graph of $f(x)$ is made of two rays and one segment connecting at points $(-1, 1)$ and $(2, -1)$, and it also passes through $(3, 1)$. Find the value of e.

4. Sketch the graph of
$$4\,||x - 5| - 3| + 3|y + 1| = 12.$$
Find the perimeter and the area of the region enclosed by the graph.

5. In the coordinate plane, let $\mathcal{R}$ denote the region consisting of points (x, y) such that
$$(|x| + |3y| - 6)\,(|3x| + |y| - 6) \leq 0.$$
What is the area of region $\mathcal{R}$?

5.4 Systems of logarithm equations (part 2)

1. The system of equations

$$\begin{aligned}\log_{10}(2000xy) - (\log_{10} x)(\log_{10} y) &= 4,\\ \log_{10}(2yz) - (\log_{10} y)(\log_{10} z) &= 1,\\ \log_{10}(zx) - (\log_{10} z)(\log_{10} x) &= 0.\end{aligned}$$

has two solutions (x_1, y_1, z_1) and (x_2, y_2, z_2). Find $y_1 + y_2$.

2. Let

$$S_1 = \{(x, y) \mid \log_{10}(1 + x^2 + y^2) \le 1 + \log_{10}(x + y)\}$$

and

$$S_2 = \{(x, y) \mid \log_{10}(2 + x^2 + y^2) \le 2 + \log_{10}(x + y)\}.$$

What is the ratio of the area of S_2 to the area of S_1.

3. Compute all values of b for which the following system has a solution (x, y) in real numbers:

$$\begin{aligned}\sqrt{xy} &= b^b,\\ \log_b\left(x^{\log_b y}\right) + \log_b\left(y^{\log_b x}\right) &= 4b^4.\end{aligned}$$

4. Let x, y, and z be positive real numbers that satisfy

$$2\log_x(2y) = 2\log_{2x}(4z) = \log_{2x^4}(8yz) \neq 0.$$

   (a) Find $\log_2 x$.

   (b) Find the general solution to this system of equations.

5. Determine all real number k such that $\log(kx) = 2\log(x + 1)$ has exactly one real solution in x.