1.8 Linear graphs and convex regions (part 1)

1. Victor's tennis club membership costs him \$200 per year and he have to pay an additional \$3 per hour in court rental fees. The town courts cost a flat fee of \$6 per hour to rent.

   (a) Write an equation that expresses Victor's total cost, C, of playing h hours of tennis in a given year at my tennis club.

   (b) Write an equation that expresses Victor's total cost, T, of playing h hours of tennis in a given year at town courts.

   (c) For how many hours of tennis should Victor play so it is worth to keep his membership at his tennis club?

   (d) Draw the graph of *piecewise function*

   $$C = \begin{cases} 6h, & 0 \le h \le \frac{200}{3}, \\ 200 + 3h, & \frac{200}{3} \le h. \end{cases}$$

   How does this graph relate to Victor's total cost of paying tennis?

   (e) Draw a graph relate to Victor's average cost of paying tennis per hour.

2. Let $X = (3, 5)$, $Y = (7, -11)$, $Z = (-13, -17)$.

   (a) Write an point-slope form equation for each of the lines XY, YZ, ZX;

   (b) Find a_1 and b_1 such that points $P_1 = (a_1, 2)$ and $P_2 = (-3, b_1)$ lie on line XY.

   (c) Find a_2 and b_2 such that points $Q_1 = (a_2, -4)$ and $Q_2 = (5, b_2)$ lie on line YZ.

   (c) Find a_3 and b_3 such that points $R_1 = (a_3, -5)$ and $R_2 = (-6, b_3)$ lie on line ZX.

3. Lines XY, YZ, ZX cut the coordinate plane into seven regions. Locate each of the points $A = (0, 0)$, $B = (1, 12)$, $C = (-7, -7)$, $D = (-2, -13)$, $E = (11, -10)$, $F = (5, 7)$, $G = (-16, -16)$ into these seven regions.

4. In a coordinate plane, shade the region that consists of all points that have positive x- and y-coordinates whose sum is less than 5. Write a system of three inequalities that describes this region.

5. Consider the region defined by the system of inequalities

   $$x + y \ge 2, \quad 0 \le x \le 5, \quad 0 \le y \le 7.$$

   Find area of the region. How many lattice points lie in the interior of the region? How many lattice points lie on the boundary of the region.

1.10 Long division for polynomials (part 1)

1. Simplify the following expressions.

   (a) $\dfrac{6x^2y}{7} \cdot \dfrac{35xy}{9x^3}$ (b) $\dfrac{x^2}{x+5} \cdot \dfrac{x^2+7x+10}{x}$ (c) $\dfrac{2-x}{x-7} \div \dfrac{x^2+5x-14}{4x-28}$

2. Note that fraction $\dfrac{14897}{123}$ is improper, and can be converted into the mixed number $121\dfrac{14}{123}$ as shown below on the left-hand side. Likewise, the fraction $\dfrac{x^4+4x^3+8x^2+9x+7}{x^2+2x+3}$ is an example of an *improper fraction*, and that the *long-division* process (illustrated below) can be used to put this fraction into the *mixed form* $x^2+2x+1+\dfrac{x+4}{x^2+2x+3}$. This process is shown below on the right-hand side.

$$
\begin{array}{rr}
 & 121 \\
123 \,\big) & \overline{14897} \\
-) & 12300 \\ \hline
 & 2597 \\
-) & 2460 \\ \hline
 & 137 \\
-) & 123 \\ \hline
 & 14
\end{array}
\qquad\qquad
\begin{array}{rr}
 & x^2+2x+1 \\
x^2+2x+3 \,\big) & \overline{x^4+4x^3+8x^2+9x+7} \\
-) & x^4+2x^3+3x^2+0x+0 \\ \hline
 & 2x^3+5x^2+9x+7 \\
-) & 2x^3+4x^2+6x+0 \\ \hline
 & x^2+3x+7 \\
-) & x^2+2x+3 \\ \hline
 & x+4
\end{array}
$$

Long-division process can also deal with zero and negative coefficients. For example, as shown below, we can convert the improper fraction $\dfrac{x^4-2x^2-3x+1}{x^2+2x-2}$ into the mixed form $x^2-2x+4-\dfrac{15x-9}{x^2+2x-4}$.

$$
\begin{array}{rr}
 & x^2-2x+4 \\
x^2+2x-2 \,\big) & \overline{x^4+0x^3-2x^2-3x+1} \\
-) & x^4+2x^3-2x^2+0x+0 \\ \hline
 & -2x^3+0x^2-3x+1 \\
-) & -2x^3-4x^2+4x+0 \\ \hline
 & 4x^2-7x+1 \\
-) & 4x^2+8x-8 \\ \hline
 & -15x+9
\end{array}
$$

Use long division process to express the following improper fractions into mixed form.

   (a) $\dfrac{2x-3}{x+1}$ (b) $\dfrac{x^2}{x-1}$

3. Use long division to show that $x^5 + x^4 + 1$ and $x^5 + x + 1$ are divisible by $x^2 + x + 1$. Verify that $x^2 + x + 1$ also divides the difference between $x^5 + x^4 + 1$ and $x^5 + x + 1$.

4. Plot points (for each of x such that $x = 10, 3, 2, 1.5, 1.1, 1, 0.9, 0.5, 0, -1, -2, -10$) to sketch the graph $\mathcal{C}$ of
$$y = \frac{x^2}{x-1}.$$

5. (Continuation) Two graphs are *asymptotic* if they become indistinguishable as the plotted points get further from the origin. (Either graph is an asymptote for the other.) Identify all asymptotic behavior of the graph $\mathcal{C}$.

1.20 The first look at quadratic functions and parabolas (part 2)

1. Graph the equations on the same system of axes: $y = x^2$, $y = 0.5x^2$, $y = 2x^2$, and $y = -x^2$. What is the effect of a in equations of the form $y = ax^2$?

2. Avery and Sasha were comparing parabola graphs on their calculators. Avery had drawn $y = 0.001x^2$ in the window $-1000 \le x \le 1000$ and $0 \le y \le 1000$, and Sasha had drawn $y = x^2$ in the window $-k \le x \le k$ and $0 \le y \le k$. Except for scale markings on the axes, the graphs looked exactly the same! What was the value of k?

3. You have seen that the graph of any quadratic function is a parabola that is symmetrical with respect to a line called the *axis of symmetry*, and that each such parabola also has a lowest or highest point called the vertex. Sketch a graph for each of the following quadratic functions. Identify the coordinates of each vertex and write an equation for each axis of symmetry.

   (a) $y = 2x^2 - 5$ (b)$y = 5 - x^2$ (c) $y = x^2 + 5x$ (d) $y = x^2 - 2x - 3$

4. When asked to solve the equation $(x - 3)^2 = 11$, Jess said, "That's easy just take the square root of both sides." Perhaps Jess also remembered that 11 has two square roots, one positive and the other negative. What are the two values for x, in exact form?

   However, when asked to solve the equation $x^2 - 8x = 1$, Jess said, "Hmm ... not so easy, but I think that adding something to both sides of the equation is the thing to do." This is indeed a good idea, but what number should Jess add to both sides? How is this equation related to the previous one?

5. Using a driver on the 8th tee, which is on a plateau 10 meters above the level fairway, Dale hits another fine shot. Explain why the quadratic function $y = 10 + 0.5x - 0.002x^2$ describes this parabolic trajectory. Why should you expect this tee shot to go more than 250 meters? To find the length of the shot, follow these steps:

   (a) Set y equal to 0 and solve for x. Explain why, and show how to arrive at $x^2 - 250x = 5000$.

   (b) Add 125^2 to both sides of this equation. Why was this number chosen?

   (c) Complete the solution and find the length of the shot.

   (d) Comment on the presence of the number 125 in the answer. What is its significance?

2.4 Algebra practice set 5

1. Given that $\dfrac{1+\sqrt{x}}{-\sqrt{x}} = y$, express x in terms of y.

2. Ivan has a $20'' \times 20'' \times 20''$ gift box that needs to be placed carefully into a $2' \times 2' \times 2'$ shipping carton, surrounded by packing peanuts. Ivan uses 1-cubic-foot bags of peanuts. He opens one bag of peanuts and spreads them evenly on the bottom of the shipping carton. Then he centers the square base of the gift box on the peanut layer, pours in another bag of peanuts, and spreads them around evenly. Now how deep are the peanuts?

3. (Continuation) Ivan pours in another m bag of peanuts, and spreads them around evenly to just cover the gift box. What is m? Can the peanuts in the m^{th} bag be all filled into the carton box? If *yes*, find the dimensions of the empty space in the resulting shipping carton; if *no*, find all volume of the left over peanuts.

4. Simplify each of the following expressions.

   (a) $\sqrt{8-2\sqrt{15}}$ (b) $\sqrt{24-8\sqrt{5}}$ (c) $\sqrt{29+12\sqrt{5}}$ (d) $\sqrt{26+2\sqrt{35}}$

5. Sketch the graph of $y = ||x-3|-5||$. If the equation $||x-3|-5|| = c$ has exactly three solutions (in x). What is c?

3.3 Work, distance and motion (part 4)

1. Bus A is 150 miles due east of Bus B. Both buses start driving due west at constant speeds at the same time. It takes Bus A 10 hours to overtake Bus B. If they had started out at the same time, had driven at the same constant speeds, but had driven toward one another, they would have met in 2 hours. What is the speed, in miles per hour, of Bus A?

2. A Prep set out to bicycle from Exeter to the beach, a distance of 10 miles. After going a short while at 15 miles per hour, the bike developed a flat tire, and the trip had to be given up. The walk back to Exeter was made at a dejected 3 miles per hour. The whole episode took 48 minutes. How many miles from Exeter did the flat occur?

3. Yan is somewhere between his home and the stadium. To get to the stadium he can walk directly to the stadium, or else he can walk home and then ride his bicycle to the stadium. He rides 7 times as fast as he walks, and both choices require the same amount of time. What is the ratio of Yan's distance from his home to his distance from the stadium.

4. Abe can paint the room in 15 hours, Bea can paint 50 percent faster than Abe, and Coe can paint twice as fast as Abe. Abe begins to paint the room and works alone for the first hour and a half. Then Bea joins Abe, and they work together until half the room is painted. Then Coe joins Abe and Bea, and they work together until the entire room is painted. Find the number of minutes after Abe begins for the three of them to finish painting the room.

5. After a soccer game, the two teams line up and march in opposite directions saying "good game" to each other. Team PEA has 20 players and team Sea Coast United has 18 players. Suppose that the team PEA line is moving twice as fast as the team Sea Coast United line does and it takes any given Sea Coast United player 12 seconds to go through the whole team PEA line, how long does it take any given PEA player to go through the whole team Sea Coast United line?