

# Lectures on Challenging Mathematics

## Essential Computational Mathematics Volume 2.4

### PC2 Number Sense

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*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596-1650)

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## 1.6 Essential number sense practices (part 6)

1. At a picnic of at least three people, a plate of 26 cookies is passed around. Each person takes one cookie and passes the plate to the next person. Jana gets the first cookie and the next to last cookie. What is the median of all possible numbers of people at the picnic?
2. Expand  $(a + b + c)(x + y + z)$ ,  $(a + b + c)(a + b + c)$ , and evaluate

$$11^2 + 14^2 + 25^2 + 2 \cdot 11 \cdot 14 + 2 \cdot 14 \cdot 25 + 2 \cdot 25 \cdot 11.$$

3. Explain why the value of  $(1 + 2 + 4)(1 + 3 + 9)$  is sum of all the positive divisors of 36. Find the sum of all the positive divisors of
  - (a) 100
  - (b) 1000
  - (c) 600
  - (d) 5929
4. Sean bought some 23 cent stamps and some 41 cent stamps and the total cost was \$9.63. How many total stamps did Sean buy?
5. A set of three distinct numbers has a greatest common divisor of 15 and a least common multiple of 8775. If the three numbers are written in order from least to greatest, what is the greatest possible value of the middle of the three numbers?

## 1.9 Essential number sense practices (part 9)

1. What is the sum of the odd positive divisors of  $(6^3 + 6^3 + 6^3 + 6^3 + 6^3 + 6^3) \cdot 10!$ .
2. When General Han counts the soldiers in his army, he uses the following method. He orders them to line up in rows of 11, then in rows of 13, and finally in rows of 17, and each time he counts the number of soldiers not in a row. One morning, he finds that there are 3 soldiers left when the rest are in rows of 11, 4 soldiers left when the rest are in rows of 13, and 9 soldiers left when the rest are in rows of 17. He knows that there are 1000 soldiers in his army. How many of the soldiers are present this morning?
3. The 9-digit number 987a56b91 is a multiple of 99 for some pair of digits  $a$  and  $b$ . What is  $ab$ ?
4. The units digit of a three-digit number is 6. What is the probability that the number is divisible by 6? Express your answer as a common fraction.
5. What is the smallest value of  $n$  for which  $n!$  ends in 50 consecutive zeros?