Lectures on Challenging Mathematics

Essential Computational Mathematics
Volume 2.3

PC2 Geometry

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"Cogito ergo Sum" – "I think, therefore I am"

René Descartes (1596-1650)

Contents

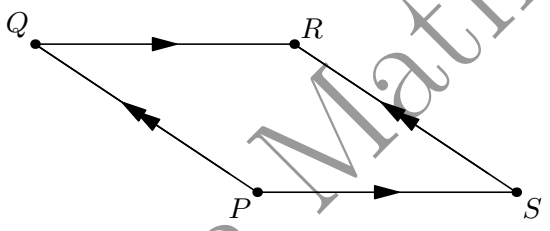
1.4 Angles (part 2)

1. The angles of a triangle are in arithmetic progression. If one of the angles is 72°, what are the measures of the other two angles? What if one of the angles is 86°?

   Complete the sentence:

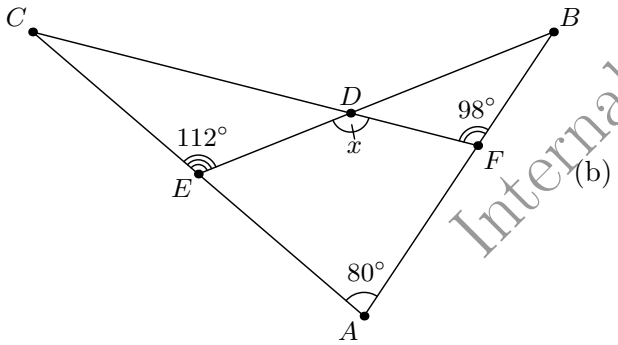
   If angles of a triangle form an arithmetic sequence, then the middle term is _____.

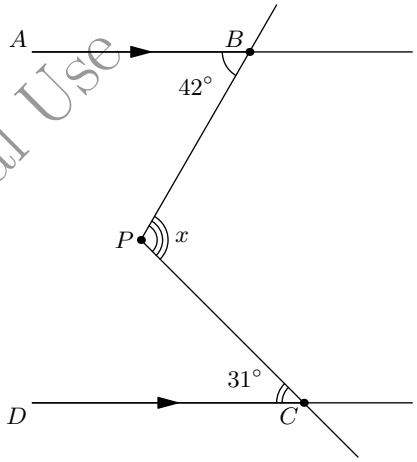
2. In parallelogram $PQRS$, the measure of $\angle P$ is four times the measure of $\angle Q$. Find the measures of all angles in the parallelogram.



3. In a hexagon $ABCDEF$ opposite sides are parallel: $AB \parallel DE$, $BC \parallel EF$, and $CD \parallel FA$. Given that $\angle A = 90°$, $\angle B = 150°$, find the measures of the other hexagon angles.

4. Find the unknown angle x in the diagrams below:

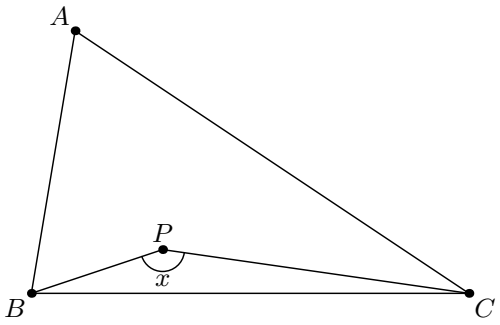




5. Point P lies inside triangle ABC. Given that $\angle A = a$, $\angle PBA = b$, and $\angle PCA = c$. Show that $\angle BPC = a + b + c$.

   (a) drawing segment AP and computing angles $\angle APB$ and $\angle APC$;

   (b) drawing lines passing through P and parallel to AB and AC.



1.10 Congruence of triangles (part 3)

1. Equilateral triangle ABX and a square $ACYZ$ are erected outwards on the sides of triangle ABC. Mark all congruent segments in the diagram. Given that $\angle BAC = 105°$, find with explanation a segment that is congruent to segment BC.

2. Let ABC be an isosceles triangle with $AB = AC$. A line parallel to the side BC intersects sides AB and AC in points D and E, respectively. Denote by M the midpoint of DE. Show that BMC is an isosceles triangle. You proof should follow a similar template that was shown for the previous problem. There are many details and each of them should be proved.

3. One more way to conclude that two triangles are congruent is

   SSS Congruence: If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.

   Let us show that why SSS congruence implies that two triangles are congruent.

   Suppose triangles ABC and DEF satisfy SSS congruence: $AB = DE$, $BC = EF$, and $CA = FD$. Arrange triangles in the plane so that vertex E coincides with vertex B and vertex F coincides with vertex C, and points A and D are on the opposite sides of the segment BC. Draw segment AD and then show that $\angle BAC = \angle BDC$. Why do triangles ABC and DEF can be considered congruent?

4. A *rhombus* is quadrilateral whose four sides all have the same length. Show that the opposite sides in a rhombus are parallel to each other.

   Start your proof as follows:

   > Let $ABCD$ be a rhombus, with $AB = BC = CD = DA$. We would like to show that line AB is parallel to line CD.
   > Triangle _____ is congruent to triangle _____ because ....

5. (Continuation) Use congruence conditions to show that in a rhombus diagonals are perpendicular to each other.

2.3 Angles (part 4)

1. Let $ABCDE$ be a regular pentagon.
   (a) Find the measure of $\angle A$.
   (b) Find the angle between diagonals AC and BD.
2. Equilateral triangle ABX is constructed outwards on the side of a square $ABCD$. Find all interior angles in the star pentagon $ABDXC$.
3. The angles of a quadrilateral $ABCD$, the degree measures of $\angle A$, $\angle B$, $\angle C$, $\angle D$ are in arithmetic progression (in this order). Given that $\angle A = 15°$. Show that two opposite sides of $ABCD$ are parallel to each other.
4. Let $ABCDEF$ be a regular hexagon.
   (a) If S is the intersection of AB and CD. Show that triangle SAD is equilateral.
   (b) Find the angle between diagonals AD and BF.
5. In a star pentagon $ABCDE$ all of its angles are equal, find the equal angle. (Hint: mark unknown angle as x, then use well the diagram in front of you.