3.6 Pythagorean theorem (part 3)

1. The sides of a right triangle are in arithmetic progression: $x-d$, x, $x+d$. Find the ratio of the smallest side to the hypotenuse.

2. Triangle ABC is right with $\angle A = 90°$. If $AB = 5$ and $AC = 12$, find length of the altitude from A onto BC.

3. You are given an isosceles triangle ABC with $AB = AC = 25$ and $BC = 14$. Find the area of triangle ABC.

4. Given an isosceles triangle with side lengths a, a, and b. Write down the condition(s) that must hold for a and b. Express the area of triangle in terms of a and b.

5. Wes and Kelly decide to test their new walkie-talkies, which have a range of six miles. Leaving from the spot where Kelly is standing, Wes rides three miles east, then four miles north. Can Wes and Kelly communicate with each other? What if Wes rides another mile north? How far can Wes ride on this northerly course before communication breaks down?

3.7 Circles (part 2)

1. The circumference of a circle is the distance around it. If a circle has radius r its circumference is equal to $2\pi r$ and it area is equal to πr^2.

   A wheel of radius 5 is rolled along the x-axis to the right without slipping until it reaches its original orientation for the first time. Find the length of the path that the center of the wheel travels.

2. Points A and B lie on a unit circle (circle with radius 1) such that $\widehat{AB} = 36°$. Find the area of the region enclosed between the chord AB and the minor arc $\widehat{AB}$.

3. Tom and Jerry start running around the unit circle in the same direction. Jerry's speed is 1 unit per minute and Tom is 1.5 faster than Jerry. How much time will pass till Tom catches Jerry? What distance will Jerry run till the moment it happens? What distance will Tom run till that moment?

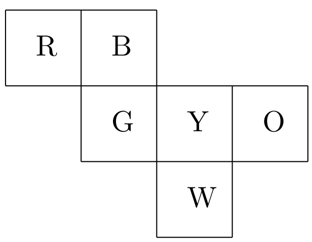
4. An equilateral triangle ABC is inscribed in a unit circle. Find which area is larger: the area of triangle ABC or the area outside triangle ABC but inside the circle.

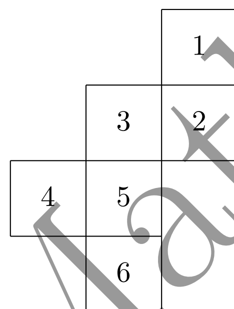
5. A standard medium pizza is 12 inches and a standard large pizza is 14 inches in diameter. In percentage terms, how much bigger is large pizza than the medium one? Suppose the width of the pizza's crust is 1 inch. Predict which pizza has a larger percent of the crust and then find which one it is.

3.8 3-D vision

1. Square corners, 5 units on a side, are removed from a 20 unit by 30 unit rectangular sheet of cardboard. The sides are then folded to form an open box. What is the surface area, in square units, of the interior of the box?

2. Six squares are colored, front and back, (R = red, B = blue, O = orange, Y = yellow, G = green, and W = white). They are hinged together as in the left-hand side figure shown below, then folded to form a cube. What color is the face opposite the white face?





3. When the net of six squares, the right-hand figure shown above, is cut out and folded to form a cube, what is the product of the numbers on the four faces adjacent to the one labeled with a 1?

4. A *tetrahedron*, also called a *triangular pyramid*, is a figure in space that has four vertices, six edges and four triangular faces. Draw a tetrahedron. The midpoints of edges of a tetrahedron form a geometric figure in space. Find how many vertices, edges, and faces it has.

5. *Pentominoes* are figures formed by joining five congruent unit squares along the edges so that each square is adjacent to (at least one) another square by sharing a common edge.

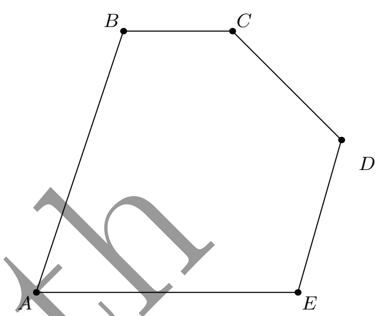
   (a) There are only 12 incongruent pentominoes, draw each one of them.

   (b) If each pentomino was cut and folded along the adjacent edges, some would form an open cube. Find each one of them.

3.9 Angles (part 5)

1. The angle-bisector of $\angle B$ in triangle ABC intersects AC in point P. The line passing through P and parallel to BC intersects AB in point Q. Explain why $BQ = PQ$.

2. In a pentagon $ABCDE$, shown in the diagram to the right,
$$\angle C + \angle D + \angle E = 360^\circ.$$
Prove that AE is parallel to BC by
   (a) showing that angles $\angle A$ and $\angle B$ are supplementary.
   (b) constructing a line through D parallel to BC.



3. In a hexagon $ABCDEF$ all of its angles are equal. Find the equal angle. Explain why the opposite sides of the hexagon are parallel.

4. In quadrilateral $ABCD$, $\angle B = 60^\circ$ and $\angle C = 80^\circ$. There exist a point X on BC such that $AX \parallel CD$ and $DX \parallel AB$. Find the measure of $\angle AXD$.

5. In a quadrilateral $ABCD$, the angle-bisectors of angles $\angle A$ and $\angle B$ intersect at P. Show that if $AD \parallel BC$ then $\angle APB = 90^\circ$. Write the converse of this statement and prove it too.