

Lectures on Challenging Mathematics

Essential Computational Mathematics Volume 2.2

PC2 Counting

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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Chapter 1

Essential Counting Practices

1.1 Essential counting practices (part 1)

1. Mary has 8 shirts that she wants to bring on her camping trip, but her Mom tells her she can only bring 6. How many different sets of six shirts can Mary decide to bring?
2. The Jones family loves Hamburger Helper. Momma Jones picks 7 of the 10 available flavors of Hamburger Helper at the grocery store to make for dinner this week. If the family eats a different Hamburger Helper flavor each of the seven days, how many different ways can their meals be made this week?
3. For positive integer $m \geq n$, we set ${}_mP_n$ (or P_n^m) as $m(m-1)\cdots(m-n+1)$. Compute the following:
 - (a) ${}_6P_3$
 - (b) P_3^9
 - (c) $\frac{P_6^{10}}{8P_4}$
 - (d) $\frac{12P_5}{P_6^{13}}$
4. Find a compact form of ${}_{88}P_{66}$ and ${}_mP_n$ by using the *factorial* notation “!”. Note that the answers for the first two problems we did can be written in the form of ${}_mP_n$ for some positive integers m and n .
5. There are six boys and six girls in the ballroom class for beginners. Determine in how many ways they can be paired for their first dance.

1.9 Essential counting practices (part 9)

1. How many factors does 48000 have? Among these factors, how many are
 - (a) odd numbers?
 - (b) even numbers?
 - (c) perfect squares?
 - (d) perfect cubes?
2. Each vertex of the following polygons is colored in a different color. How many distinct triangles can be formed by connecting three different vertices of a
 - (a) pentagon?
 - (b) hexagon?
 - (c) heptagon?
 - (d) octagon?
3. What is the largest five-digit integer whose digits have a product equal to 5!? How many five-digit integer whose digits have a product equal to 5!?
4. What is the sum of the numbers less than 200 that have exactly 9 divisors?
5. Seven boys and nine girls applied for jobs at a park. Two boys will be hired to be ticket takers, two girls will be chosen to be life guards and 1 other kid will be chosen to work at the concession stand. In how many ways can these 5 kids be chosen?