## 1.3 Graphing linear inequalities in two variables

1. A *linear inequality* in $x$ and $y$ is an inequality that can be written as follows:

$$ax + by \;\substack{<\\ =\\ >}\; c.$$

An ordered pair $(x, y)$ is a *solution* of a linear inequality if the inequality is true when the values of $x$ and $y$ are substituted into the inequality.

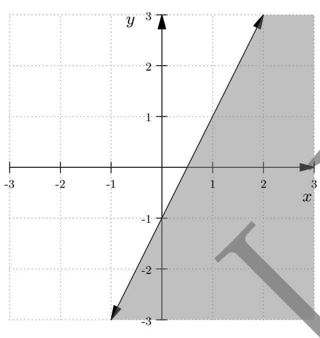
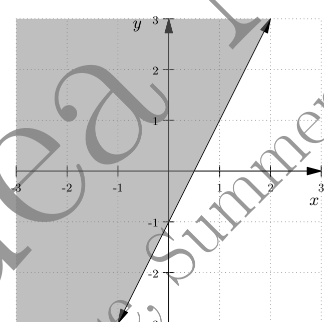
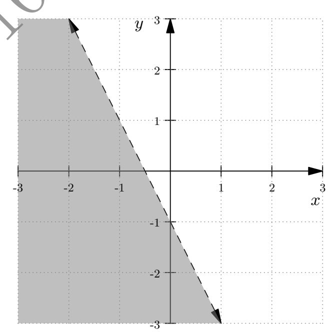
The *graph* of a linear inequality in two variables is the graph of the solutions of the inequality. In the graph, a *dashed* or *interrupted* line is used for inequalities with $>$ or $<$ to show that the points on the line are not solutions, and a *solid* line for inequalities with $\geq$ or $\leq$ to show the points on the line are solutions.

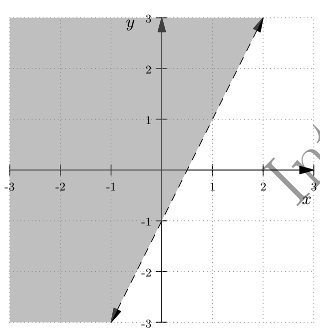
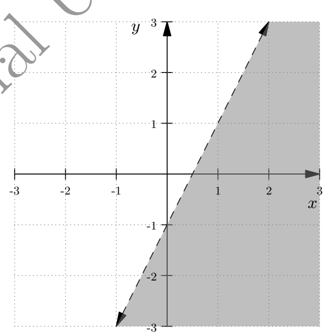
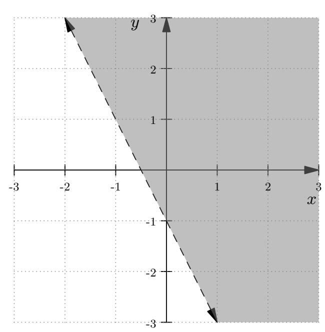
Check whether the ordered pair is a solution to $2x - 3y \geq -2$:

(a) $(0, 0)$ (b) $(0, 1)$ (c) $(2, -1)$ (d) $(2, 2)$

2. Match the inequality with its graph:

A. $2x - y \leq 1$ B. $-2x - y < 1$
C. $2x - y \geq 1$ D. $2x - y > 1$
E. $2x - y < 1$ F. $-2x - y > 1$

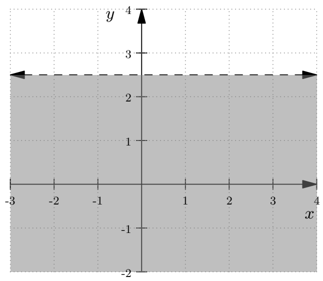
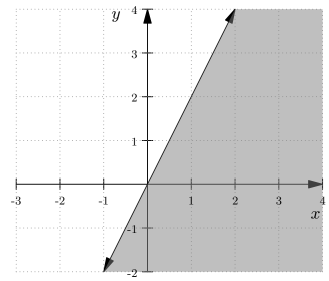
(a) 
(b) 
(c) 

(d) 
(e) 
(f) 

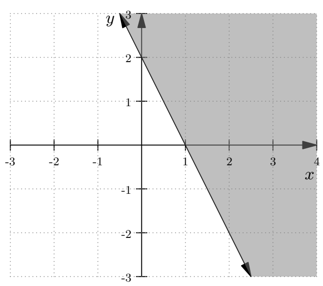
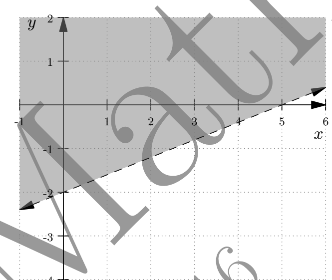
3. Sketch the graph of the inequality:

(a) $x < -2$ (b) $y \leq 1$ (c) $x + y > 3$ (d) $2x - 3y \geq -2$

4. Write an inequalities whose solutions are shown in the graphs:

(a)  (b) 

5. Write an inequalities whose solutions are shown in the graphs:

(a)  (b) 

6. Lee's pocket change consists of $x$ quarters and $y$ dimes. Put a dot on every lattice point $(x, y)$ that signifies that Lee has exactly one dollar of pocket change. What equation describes the line that passes through these points? Notice that it does not make sense to connect the dots in this context, because $x$ and $y$ are *discrete* variables, whose values are limited to nonegative integers.

7. (Continuation) Put a dot on every lattice point $(x, y)$ that signifies that Lee has at most one dollar in pocket change. How many such dots are there? What is the relationship between Lee's change situation and the inequality $0.25x + 0.10y \leq 1.00$?

8. (Continuation) Write two inequalities that stipulate that Lee cannot have fewer than zero quarters or fewer than zero dimes.

9. You are on a treasure-diving ship that is hunting for gold and silver coins. Objects collected by the divers are placed in a wire basket. One of the divers signals you to reel in the basket. It feels as if it contains no more than 50 pounds of material. If each gold coin weights about 0.5 ounces and each silver coin weighs about 0.25 ounces, what are the different amounts of coins that could be in the basket?

10. Find area of the intersection between graphs of $|x| < 3$ and $|y| < 2$.

## 1.4 More operations rules with (integer) exponents

1. The product $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ is usually abbreviated by 9!, and read *9 factorial*. Because factorials are often large, they are a challenge to compute and display. Evaluate $n!$ for $n = 1, 2, \ldots, 8$.

2. Find a pair of positive integers $(m, n)$ such that $20 \cdot 19 \cdot 18 \cdots 14 = \dfrac{m!}{n!}$.

3. Express $4 \cdot \left(2^3 + 2^3\right)\left(3^5 + 3^5 + 3^5\right)$ in form of $a^b$ and find the minimum possible value of $a + b$, where $a$ and $b$ are positive integers.

4. Determine the number of ways to assign positive integers to the heart, diamond, and club so that the identity $x^{\heartsuit} x^{\diamondsuit} x^{\clubsuit} = x^7$ holds.

5. Rewrite each of the following as a single fraction.

   (a) $3a + 5a^{-1} + 7a^{-2}$

   (b) $-f^{-1} + f - 2f^{-2} + 3f^3 - 5f^{-5} + 8f^8$

   (c) $ab^{-1} + ba^{-1}$

   (d) $a^{-1} + b^{-1} + c^{-1}$

   (e) $(k\ell)^{-1} + (\ell m)^{-2} + (mn)^{-3}$

   (f) $\left(a^{-1} + b^{-1} + b^{-2}\right)^{-1}$

6. Find positive integers $a, b, c, d, e$ such that each of $2^a, 3^b, 4^c, 8^d, 9^e$ divides 20! while none of $2^{a+1}, 3^{b+1}, 4^{c+1}, 8^{d+1}, 9^{e+1}$ divides $n = 20!$? How about $n = 60!$?

7. The decimal representation of 20! ends in a bunch of zeros. How many zeros? How about 60!?

8. Find a pair $(a, n)$ of positive integers such that $2^a \cdot n! = 2 \cdot 4 \cdot 6 \cdot \cdots \cdot 24$.

9. Find the least positive integer $m$ such that $m!$ is a multiple of 20!14! (the product of 20! and 14!).

10. Find a triple $(b, p, q)$ of positive integers such that

$$\frac{p!}{2^b \cdot q!} = 1 \cdot 3 \cdot 5 \cdots 51.$$

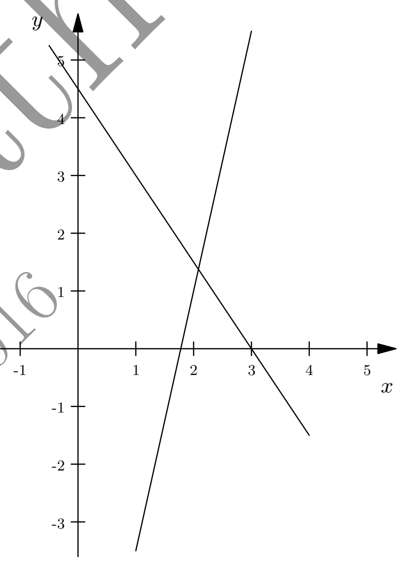
## 1.5 Solving linear systems by substitution or by elimination

1. Make an attempt to solve the system of equations using graphing method:

$$\begin{cases} 4x + 3y &= 2, \\ 8x - 9y &= -1. \end{cases}$$

What difficulties do you encounter? Verify if a pair $(\frac{1}{4}, \frac{1}{3})$ is a solution to the system. Explain why it is easy to check if a pair is a solution, but it is hard to find such pair using the graphing method.

2. The figure at right shows the graphs of two lines.




(a) Use the graphs (the axis markings are one unit apart) to estimate the coordinates of the point that belongs to both lines.

(b) The *system of equations* that has been graphed is

$$\begin{cases} 9x - 2y = 16, \\ 3x + 2y = 9. \end{cases}$$

Jess took one look at these equations and knew right away what to do. "Just add the equations and you will find out quickly what $x$ is." Follow this advice, and explain why it works.

3. (Continuation) Find the missing $y$-value by inserting the $x$-value you found into either of the two original equations. Do the coordinates of the intersection point agree with your estimate? These coordinates are called a *simultaneous solution* of the original system of equations. Explain the terminology.

4. Draw the lines $4x + 3y = 20$ and $y = 2x - 2$. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20, \\ y = 2x - 2. \end{cases}$$

Dale took one look at these equations and offered a plan: "The second equation says you can *substitute* $2x - 2$ for $y$ in the first equation. Then you have only one equation to solve." Explain the logic behind Dale's substitution strategy. Carry out the plan, and compare the exact coordinates of the intersection point with your estimates.

5. Solve the following linear system by substitutions:

$$\begin{cases} -x + y = 1, \\ 2x + 3y = -2. \end{cases}$$

6. Draw the lines $3x+2y=6$ and $3x-4y=17$. First use the figure to estimate the coordinates of the point that belongs to both lines. Second consider the system of equations

$$\begin{cases} 3x + 2y = 6, \\ 3x - 4y = 17. \end{cases}$$

Randy took one look at these equations and knew right away what to do. "Just subtract the equations and you will find out quickly what $y$ is." Follow this advice.

7. (Continuation) Find the missing $x$-value by inserting the $y$-value you found into one of the two original equations. Does it matter which one? Compare the intersection coordinates with your estimate.

8. (Continuation) If you *add* the two given equations, you obtain the equation of yet another line. Add its graph to the figure. You should notice something. Was it expected?

9. Draw the lines $4x+3y=20$ and $3x-2y=-5$. Use the figure to estimate the coordinates of the point that belongs to both lines. The system of equations is

$$\begin{cases} 4x + 3y = 20, \\ 3x - 2y = -5. \end{cases}$$

Lee took one look at these equations and announced a plan: "Just multiply the first equation by 2 and the second equation by 3." What does changing the equations in this way do to their graphs? Lee's plan has now created a familiar situation. Do you recognize it? Complete the solution to the system of equations. Do the coordinates of the point of intersection agree with your initial estimate?

10. Solve each of the systems of equations using elimination:

(a) $\begin{cases} 3x + 4y = 1 \\ 4x + 8y = 12 \end{cases}$ (b) $\begin{cases} 2x + 3y = -1 \\ 6x - 5y = -7 \end{cases}$

## 1.6 Applications of linear systems

1. Farmer MacGregor wants to know how many cows and ducks are in the meadow. He counted 56 legs and 17 heads. How many cows and ducks are there?

2. A store sold 28 pairs of cross-trainer shoes for a total of \$2220. Style A sold for \$70 per pair and Style B for \$90 per pair. How many of each style were sold?

3. At the Exeter Candy Shop, Jess bought 5.5 pounds of candy — a mixture of candy priced at \$4 per pound and candy priced at \$3.50 per pound. Given that the bill came to \$20.75, figure out how many pounds of each type of candy Jess bought.

4. Write and graph an equation that states

   (a) that the perimeter of an $l \times w$ rectangle is 768 cm;

   (b) that the width of an $l \times w$ rectangle is half its length.

   Explain how the two graphs show that there is a unique rectangle whose perimeter is 768 cm, and whose length is twice its width. Find the dimensions of this rectangle.

5. A large family went to a restaurant for a buffet dinner. The price of the dinner was \$12 for adults and \$8 for children. If the total bill for a group of 13 persons came to \$136, how many children were in the group?

6. Homer began peeling a pile of 44 potatoes at the rate of 3 potatoes per minute. Four minutes later Christen joined him and peeled at the rate of 5 potatoes per minute. When they finished, how many potatoes had Christen peeled?

7. Of the eggs produced by salmon, 80% hatch, and of those, 25% survive to migrate to the ocean. How many eggs are needed to produce 100 salmon that migrate to the ocean?

8. A club had collected an amount of money to split among the top three finishers of their annual science fair. The first place finisher will receive one-half of the money. The second place finisher will receive one-third of the money. The third place finisher will receive \$200. How much money will the first place finisher receive?

9. You are buying some cans of juice and some cans of soda for the dorm. The juice is \$0.60 per can while the soda is \$0.75. You have \$24 of dorm funds, all to be spent.

   (a) Write an equation that represents all the different combinations of juice and soda you can buy for \$24.

   (b) Is it possible to buy exactly 24 cans of juice and spend the remainder on soda? Explain.

   (c) How many different combinations of drinks *are* possible?

   (d) Find $n$ such that the line equation $4x + 5y = n$ is closely related to this problem.

10. The diagram shows a rectangle that has been divided into ten squares of different sizes. The smallest square is $3 \times 3$. What are the dimensions of the other squares?