

# Lectures on Challenging Mathematics

## Essential Computational Mathematics Volume 2.1

### PC2 Algebra

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*“Cogito ergo Sum” – “I think, therefore I am”*

René Descartes (1596-1650)

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## 1.5 Proportions and mixed solutions

1. Ten cc of a solution of acid and water is 30% acid. I wish to dilute the acid in the mixture by adding water to make a mixture that is only 6% acid. How much pure water must I add to accomplish this?
2. The cooling system of Alex's car holds 10 quarts. It is now filled with a mixture that is 60% water and 40% antifreeze. Hearing a weather forecast for severe cold, Alex decides to increase the strength of the antifreeze mixture to 50%. To do this, Alex must drain off a certain number of quarts from the cooling system and then replace them by pure antifreeze. How many quarts must be drained?
3. Taylor works after school in a health-food store, where one of the more challenging tasks is to add cranberry juice to apple juice to make a cranapple drink. A liter of apple juice costs \$0.85 and a liter of cranberry juice costs \$1.25. The mixture is to be sold for exactly the cost of the ingredients, at \$1.09 per liter. How many liters of each juice should Taylor use to make 20 liters of the cranapple mixture?
4. Let  $a, b, c, d$  be positive numbers such that

$$\frac{a}{b} \leq \frac{c}{d}.$$

Show that

$$\frac{a}{b} \leq \frac{a+c}{b+d} \leq \frac{c}{d}.$$

5. (Continuation) Why the above inequality is called the *bar tender's* inequality? Assuming  $\frac{c}{d} < 1$  could be helpful.

## 1.9 Numbers and their pairwise sums

1. Numbers  $a$ ,  $b$ ,  $c$  are labeled in that order from left to right on the number line. One tries to arrange numbers  $a + b$ ,  $b + c$ ,  $c + a$  on the number line. Considering the relative positions of these three numbers, what are the possible scenarios?
2. Numbers  $a$ ,  $b$ ,  $c$ ,  $d$  are labeled in that order from left to right on the number line. One tries to arrange numbers  $a + b$ ,  $b + c$ ,  $c + d$ ,  $a + c$ ,  $b + d$ ,  $a + d$  on the number line. Considering the relative positions of these six numbers, what are the possible scenarios?
3. A person from the audience thinks of four numbers. She tells the magician that their pairwise sums are

34, 36, 40, 42, 46, 48.

Can the magician find her numbers? What is the issue with this magic trick?

4. Five numbers are given. Their pairwise sums are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Find the sum of these numbers.
5. Ana, Barbara, and Carol want to know their average age. But no lady wants to disclose her age. Once they agree on a strategy they will fairly implement it. Is there a way to do that?