Lectures on Challenging Mathematics

Essential Computational Mathematics
Volume 1.4

PC1 Number Sense

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"Cogito ergo Sum" – "I think, therefore I am"

René Descartes (1596-1650)

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1.2 Switching the order of operations (part 1)

1. Compute $1-2+3-4+5-6+7-8+9-10+11-12+13-14+15-16+17-18+19$ and $21+22+23+24+25+26+27+28+29-(1+2+3+4+5+6+7+8+9)$.

2. In order to evaluate the sum $s = 27+29+31+33+35+37+39+41+43$, Sam did

$$\begin{array}{rrcccccccccccccccc} & s & = & 27 & + & 29 & + & 31 & + & 33 & + & 35 & + & 37 & + & 39 & + & 41 & + & 43 \\ +) & s & = & 43 & + & 41 & + & 39 & + & 37 & + & 35 & + & 33 & + & 31 & + & 29 & + & 27 \\ \hline & 2s & = & & & & & & & & & & & & & & & & & \end{array}$$

   Help Sam complete his job.

3. Compute $5+8+11+14+\cdots+122+125$.

4. How many digits are in the value of $2^{150}\times 5^{50}\div 4^{51}$?

5. Using the four integers 2, 3, 5 and 7 once each — in any order — and three arithmetic operations selected from among addition, subtraction, multiplication, and division, is it possible to write an expression whose value is 0? Using the same numbers and conditions, how many of the integers from 1 to 24 can you form? You may use parentheses.

   This is a variation of the *Game of 24*: choosing 4 random cards from a standard deck of cards, and then using the numbers on the cards once each — in any order — and three arithmetic operations selected from among addition, subtraction, multiplication, and division, to try to write an expression whose value is 24. (It is custom to consider the face values of Jack, Queen, and King as 11, 12, and 13, respectively.) This is a great game to play and it helps you to improve your arithmetic ability and number sense.

1.8 GCD and LCM (part 2)

1. Erin is having a party, and she made 63 witch cookies, 45 Frankenstein cookies, and 54 black cat cookies. She wants to make up plates of cookies to set on the tables. If each plate is to contain the same amount of each type of cookie, and there are no left over cookies, what is the greatest number of plates she can make? How many of each cookie would be on each plate?

2. What is the product of $\gcd(10, 35)$ and $\text{lcm}(10, 35)$, $\gcd(12, 42)$ and $\text{lcm}(12, 42)$, and $\gcd(26, 91)$ and $\text{lcm}(91, 26)$? What did you observe and why does your observation make sense?

3. Three numbers have a product of 2310. What is the least common multiple of the three numbers?

4. Sam, Monique, and Sandra collect spare change. Sam has collected 60 nickels, Monique has collected 48 dimes, and Sandra has collected 72 quarters. They want to make several stacks of coins such that each stack is exactly the same. What is the largest number of stacks that they can make if each stack must be exactly the same? How much money would be in each stack?

5. When is the least common multiple of two numbers equal to their product?