Lectures on Challenging Mathematics

Essential Computational Mathematics
Volume 1.3

PC1 Geometry Sense

Summer 2017

Edited by

Zuming Feng Yunhua Xu Chengde Feng Ivan Borsenco

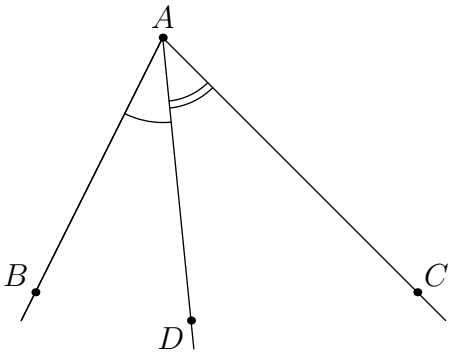
"Cogito ergo Sum" – "I think, therefore I am"

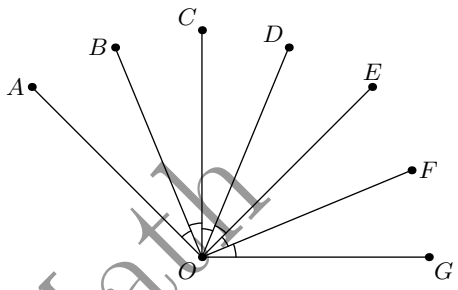
René Descartes (1596-1650)

Contents

1.6 Angles (part 2)

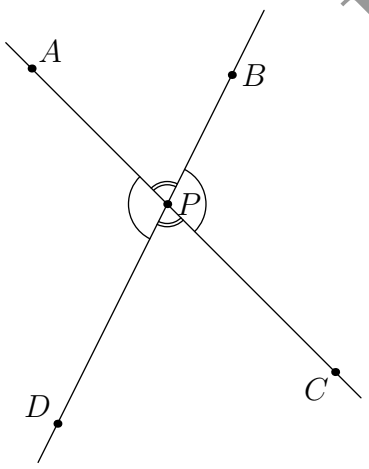
1. *Adjacent* angles are two angles in a plane that have a common vertex and a common side but no common interior points. In the diagram below, angles $\angle BAD$ and $\angle CAD$ are adjacent.

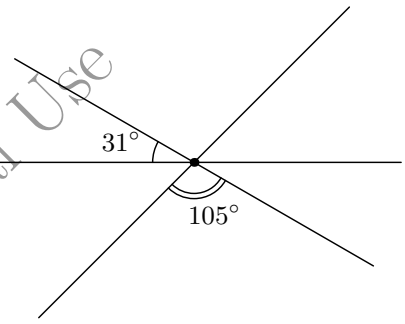




Angle $\angle AOG$ is divided by rays OB, OC, OD, OE, OF into six congruent angles. How many pairs of congruent adjacent angles are there?

2. *Vertical angles* is a pair of non-adjacent angles formed by the intersection of two straight lines. When two lines intersect they form two pairs of vertical angles. Vertical angles are congruent. For example, in the diagram below $\angle APB = \angle CPD$ and $\angle APD = \angle BPC$.





Three lines are intersecting in one point forming six angles at the intersection. Given that two of these angles are equal to 31° and 105° angles, find the measures of the remaining four angles.

3. An angle of measure between 0° and 90° is called an *acute* angle. An angle of measure between 90° and 180° is called an *obtuse* angle.

   (a) An acute angle α and an obtuse angle β are prime numbers. Given that $\alpha + \beta = 112°$, find all possible pairs (α, β) that satisfy the conditions of the problem.

(b) Let α be the measure of an angle. If we double the angle, we get an acute angle and if we triple the angle we get an obtuse angle. Find all possible values of α.

4. The *bisector* of an angle is a ray that divides the angle into two congruent neighboring angles.

   Given a right angle $\angle XOY$ and rays OZ and OW inside it such that OW is the bisector of $\angle ZOY$. If $\angle XOZ - \angle WOY = 30°$, find $\angle ZOW$. What if $\angle WOY - \angle XOZ = 30°$?

5. Consider a straight angle $\angle AOE$ and rays OB, OC, and OD dividing this angle into four angles. It is given that OB is the angle-bisector of $\angle AOC$ and OD is the angle-bisector of $\angle COE$.

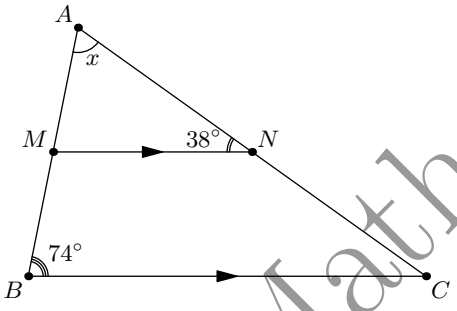
   (a) Find $\angle BOD$ assuming $\angle AOC = 40°$.

   (b) Find $\angle BOD$ assuming $\angle AOC = 108°$.

   (c) Find $\angle BOD$ choosing any size for $\angle AOC$.

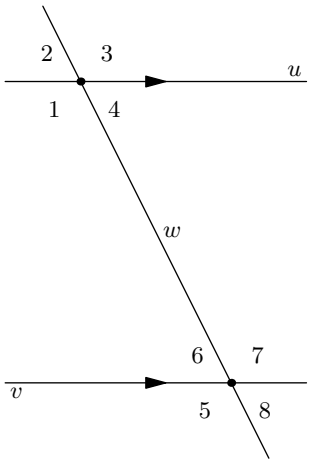
1.10 Parallel lines (part 3)

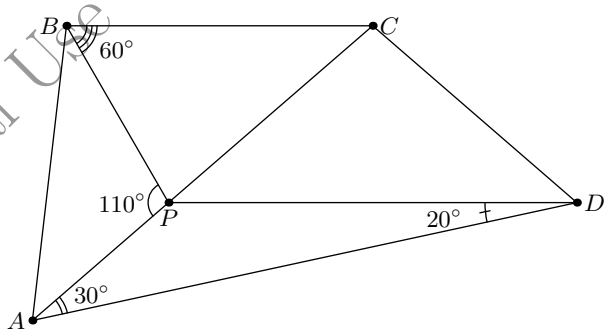
1. Points M and N are lying on the sides AB and AC of triangle ABC such that MN is parallel to BC. Given that $\angle B = 74^\circ$ and $\angle ANM = 38^\circ$, find $\angle A$.



2. **The converse of the Corresponding Angles Postulate.** If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.

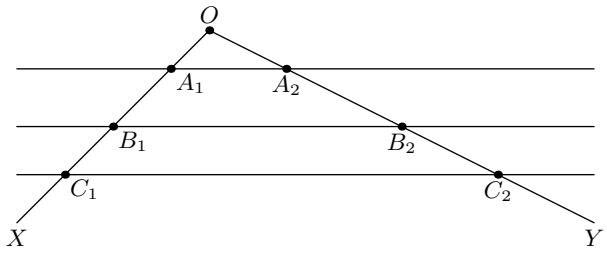
   What condition must be true about alternate interior angles to show that two lines are parallel? What condition must be true about same-side interior angles to show that two lines are parallel?





   In quadrilateral $ABCD$ point P lies on the diagonal AC such that $\angle APB = 110^\circ$, $\angle PBC = 60^\circ$, $\angle PDA = 20^\circ$, and $\angle PAD = 30^\circ$. Find as many angles as possible in the diagram. Explain why lines BC and PD are parallel.

3. Points A_1, B_1, C_1 lie on ray OX, while points A_2, B_2, C_2 lie on ray OY. Lines A_1A_2 and B_1B_2 are parallel. Lines A_1A_2 and C_1C_2 are parallel.



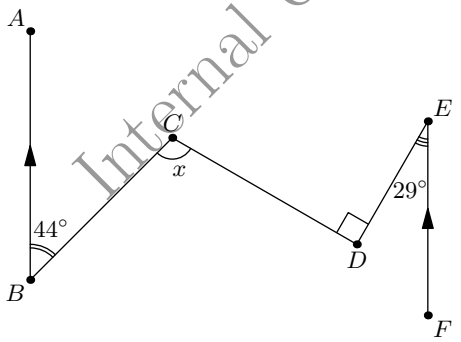
Complete the proof that explains why B_1B_2 must be parallel to C_1C_2:

> Lines A_1A_2 and B_1B_2 are parallel, so by the Corresponding Angles Postulate, $\angle OA_1A_2 =$ _____.
>
> Lines A_1A_2 and C_1C_2 are parallel, so by the Corresponding Angles Postulate, $\angle OA_1A_2 =$ _____.
>
> Therefore _____ and by the converse of the Corresponding Angles Postulate we get _____.

In general, after solving this problem, we can say that, if two lines are parallel to a third line, then they are parallel to each other.

4. In a quadrilateral all angles are equal. Explain why the opposite sides of the quadrilateral are parallel to each other. What is name of the equiangular quadrilateral?

5. Find unknown angle x in the diagram below:

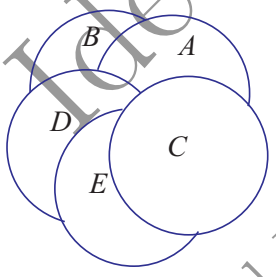


Chapter 2

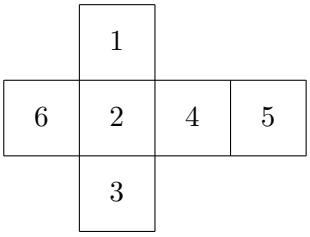
Geometry Challenges

2.1 3-D vision (part 1)

1. There are five coins placed flat on a table according to the figure in the left-hand side shown below. What is the order of the coins from top to bottom?

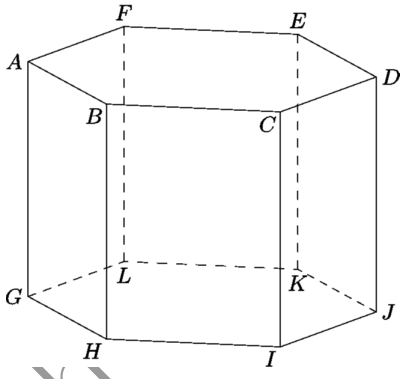


2. The figure shown below may be folded to form a number cube. Three number faces come together at each corner of the cube. What is the largest sum of three numbers whose faces come together at a corner?



3. Recall that in space two lines that do not intersect are either parallel or skew to each other.

Consider a hexagonal box drawn on the diagram to the right:



(a) Name five lines that are parallel to AG.

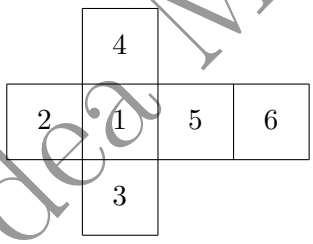
(b) Name three lines that are parallel to AB.

(c) Name four lines that are skew to AB.

(d) Name four planes parallel to FL.

(e) How many pairs of parallel planes are shown in the diagram?

4. When the net of six squares on the right is cut out and folded to form a die. When observing a die, one can see at most three faces at a time. Of the numbers $5, 6, 7, \ldots, 15$, which cannot be the sum of the observed faces?



5. The centers of all faces of a cube form a geometric solid figure in space. Use the diagram below to find how many vertices, edges, and faces it has.