Lectures on Challenging Mathematics

Essential Computational Mathematics
Volume 1.2

PC1 Counting

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"Cogito ergo Sum" – "I think, therefore I am"

René Descartes (1596-1650)

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Chapter 1

Basic Counting Practices

1.1 Basic counting practices (part 1)

1. When getting dressed in the morning, Sally must choose between 6 shirts, 4 pairs of jeans, and 8 pairs of shoes. If she chooses one of each to wear that day, how many different outfits can she choose?

2. Bob's teacher told him to solve the even problems from 18 to 64 inclusive for homework last night. How many problems was he assigned?

3. How many pairs of two whole numbers, both greater than 25, have a sum of 200? Pair $(50, 150)$ is considered different from pair $(150, 50)$.

4. Hundreds of people were standing in line to buy tickets to a concert. Each person had a wrist band with an integer marking their place in line. Starting with the 74th person in line a member of the band came out and gave every 4th person a back stage pass. If the band member had 28 back stage passes, what was the number of the person in line that got the last back stage pass?

5. Among any three students who come to the IDEA Math camp there are two who know each other. Explain why in a class with six students we can always find three who know each other.

1.6 Basic counting practices (part 6)

1. Little John bought two tickets to a movie for him and his sweetheart. Sadly, it turned out that the two seats they got are not next one to another, but there are eleven empty seats between them! During each commercial John can move two or four seats closer to his sweetheart. Can he eventually sit next to her?

2. How many different seven-digit phone numbers are available if the only restriction is that the first digit cannot be 0? How many of them are even?

3. Determine the maximum number of points of intersection among

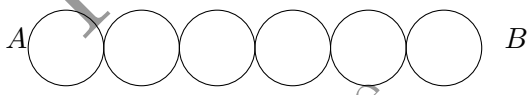
   (a) a square, a circle and a line;

   (b) two circles, a square and three lines.

   In each case use a table to simplify the count of intersection points.

4. The number π is a mathematical constant, the ratio of a circle's perimeter to its diameter. In particular, if the radius of the circle is 1, its perimeter is equal to 2π.

   Consider congruent circles of radius 1 with centers at $(1,0)$, $(3,0)$, $(5,0)$, $(7,0)$, $(9,0)$, and $(11,0)$. How many paths of length 6π are there from $A = (0,0)$ to $B = (12,0)$ if the paths are made of semicircles?



5. Bob has a choice of toppings for his ice cream. He must choose one of caramel, hot fudge, and marshmallow, two of pecans, walnuts, and almonds, and three of strawberry, M&Ms, Butterfinger, sprinkles, and Snickers.

   (a) How many different ice cream cones could he get?

   (b) Bob decides that he does not want to get both walnuts and caramel at the same time. How many different ice cream cones could Bob get?