Samples of Contest Math 4

I Algebra

1. Two days ago, *x* dogs skated. Yesterday, exactly 20% more than *x* dogs skated. Exactly 40% more dogs skated today than skated yesterday. What is the minimum positive integer value of *x*?

2. In a 10-km race, First runner beat Second runner by 2 km, and First Runner beat Third Runner by 4 km. If all three runners always ran at constant rates, by how many km did Second Runner beat Third Runner?

3. If $-1 < x < 0$, the for what positive integer $n \le 2016$ does x^n take on its least value?

4. The solutions of $x^3 - 3x^2 + kx + 15 = 0$ form an arithmetic sequence. What is the value of *k*?

5. A certain computer program will take 2000 years to run using current technology. Every year, advances in technology make it possible to run the program in half the time it would have taken starting in the previous year. However, once the program is started it cannot be interrupted to apply newer technology. Including the years spent waiting to start, what is the least number of years it will take to finish running the program? Express your answer to the nearest whole number.

6. Claudia has 12 coins, each of which is a 5-cent coin or a 10-cent coin. There are exactly 17 different values that can be obtained as combinations of one or more of her coins. How many 10-cent coins does Claudia have?

7. On their vacation, the Ship family took a boat ride to Shipwreck Island. The table shows the fees charged by the boat company, based on each passenger's age. The total charge for the 12 family members was $73. If the number who are "others" exceeds the number who are 62 and older, what is the maximum possible number of children ages 3 to 12?

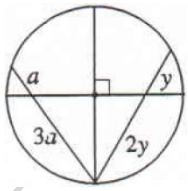
Age	Fee
2 & under	No charge
3 to 12	$3 each
62 & older	$7 each
All others	$10 each

8. Tire pressure is directly proportional to temperature on a temperature scale where zero degrees is absolute zero. Given that temperatures in degrees Celsius (*C*) and degrees Fahrenheit (*F*) are related by the formula $F = \frac{9}{5}C + 32^\circ$, and absolute zero is −273.15°C, by what percent does tire pressure decrease when the temperature drops from 80°F to 40°F? Express your answer to the nearest whole number.

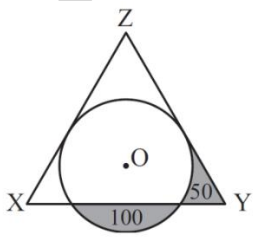
9. Four steps by Dog Kat can exactly cover the distance of 7 steps by Cat Fat. The time for Kat running 6 steps is equal to that of Fat running 5 steps. Supposing that Fat is 55 yards away before Kat starts to catch Fat, how far can Fat run before it gets caught?

II Geometry

10. Two perpendicular diameters are drawn in a circle, as shown. From an endpoint of one of them chords that intersect the other diameter at different points are drawn as shown. The horizontal diameter splits one of the chords into a 2:1 ratio and the other into a 3:1 ratio. What is the ratio, bigger to smaller, of the lengths of the two chords?



11. Circle O is tangent to two sides of equilateral triangle XYZ. If the two shaded regions have areas 50 cm^2 and 100 cm^2 as indicated, what is the ratio of the area of triangle XYZ to the area of circle O? Express your answer as a decimal to the nearest hundredth.



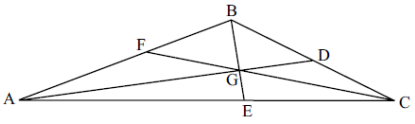
12. In the largest right triangle, the altitude to the hypotenuse has a length of 12. If every line segment has an integral length, what is the perimeter of the largest right triangle?

13. A 3-4-5 right triangle made of paper is cut along the altitude from the right angle, resulting in 2 smaller right triangles. These 2 triangles are then cut the altitudes from their right angles, then the 4 resulting right triangles are cut the same way and finally, 8 triangles are cut the same way, resulting in 16 smaller right triangles. What is the sum of the perimeters of these 16 triangles? Express your answer as a decimal to the nearest tenth.

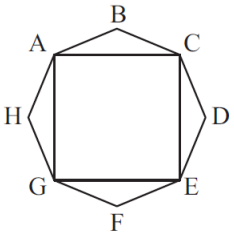
14. The *y*-intercepts, *P* and *Q*, of two perpendicular lines intersecting at the point *A*(6, 8) have a sum of zero. What is the area of ΔAPQ ?

15. A square in the coordinate plane has vertices whose *y*-coordinates are 0, 1, 4, and 5. What is the area of the square?

16. Points D, E and F lie along the perimeter of DABC such that AD, BE and CF intersect at point G. If AF = 3, BF = BD = CD = 2 and AE = 5, then what is BG/EG? Express your answer as a common fraction.



17. Octagon ABCDEFGH, shown here, has eight sides of equal length, and ACEG is a square. If the area of square ACEG is 7/9 of the area of octagon ABCDEFGH, what is the ratio of the area of square BDFH to the area of octagon ABCDEFGH? Express your answer as a common fraction.



III Number Sense

18. If $x > 0$ and I divide x by y, the quotient is 3 and the remainder is 7. If I divide y by x, the remainder is 12. What is the value of x?

19. Every coin in my piggy bank has a face value of 50¢, 25¢, 10¢, 5¢, or 1¢. The bank contains many coins of each type. At most how much money can I withdraw from piggy bank without being able to make change for \$1?

20. The number 1 can be written as a sum of n positive numbers (not necessarily distinct), each of which consists entirely of 8s, or entirely of 0s and 8s. What is the least possible value of n?

21. How many positive integers less than or equal to 100 have the same number of odd factors as even factors?

22. The product (8) (888…8), where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

23. If 738 consecutive integers are added together, where the 178th number in the sequence is 4,256,815, what is the remainder when this sum is divided by 6?

24. For the integer data set {11, 13, 15, 17, x, y}, the unique mode, median and mean form an increasing arithmetic sequence, in that order. What is the greatest possible value of y?

25. Let n be a 5-digit number, and let q and r be the quotient and the remainder, respectively, when n is divided by 100. For how many values of n is $q + r$ divisible by 11?

26. A base-10 three-digit number n is selected at random. Which of the following is closest to the probability tha the base-9 representation and the base-11 representation of n are both three-digit numerals?

27. In year N, the 300^{th} day of the year is a Tuesday. In the year $N + 1$, the 200^{th} day is also a Tuesday. On what day of the week did the 100^{th} day of the year $N – 1$ occur?

28. In a math test in Rice Middle School Math Club, the average of the boys' scores is 2 points more than the average of the scores of the whole class, and the average of the girls' scores is 1 point less than the average of the scores of the whole class. If the sum of the boys' scores is 942, and the sum of the girls' scores is 1800, find the number of the boys and the number of the girls in the class.

IV Counting & Probability

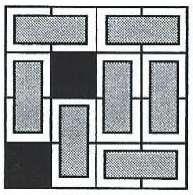
29. Balls are randomly removed from a bag without replacement. If the probability that the first 5 balls withdrawn are all green is one-half, what is the least possible number of balls in the bag at the start?

30. The numbers1, 2, 3, 4, 5, 6 are randomly written around a circle. What is the probability that there are four neighboring numbers such that the sum of middle two numbers is less than the sum of the other two?

31. How many different quadrilaterals have vertices with integer coordinates (x, y) such that $0 \le x \le 2$ and $0 \le y \le 2$?

32. Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

33. Two unit squares are removed from a 4×4 grid. Seven dominoes are available to cover each of the remaining 14 squares such that each domino covers two adjacent unit squares. One such example is shown. Including the pair shown in this example, how many pairs of unit squares can be removed so that the seven dominoes can cover the remaining 14 unit squares?



34. In the city of Trichotomy, every day the weather is exactly one of the following: sunny, cloudy or rainy. Each day has a 50% chance of having the same weather as the day before and a 25% chance of having each of the other two types of weather. If it does not rain on Friday, what is the probability that there will be no rain during the weekend (Saturday and Sunday)? Express your answer as a common fraction.

35. A room has eight switches, each of which controls a different light. Initially, exactly five of the lights are on. Three people enter the room, one after the other. Each person independently flips one switch at random and then exits the room. What is the probability that after the third person has exited the room, exactly six of the lights are on? Express your answer as a common fraction.

36. Compute the number of permutations $x_1, x_2, \cdots, x_{10}$ of the integers $-3, -2, -1, \ldots, 5, 6$ that satisfy the chain of inequalities $x_1x_2 \le x_2x_3 \le \cdots \le x_9x_{10}$.