1.9   Season 1 Episode 9, 11/22/2015

Selected problems from HMMT November, 2015

1. (Team Round P1)
   Triangle ABC is isosceles, and $\angle ABC = x°$. If the sum of the possible measures of $\angle BAC = 240°$, find $x°$.

2. (General Test P3)
   Neo has an infinite supply pf red pills and blue pills. When he takes a red pill, his weight will double, and when he takes a blue pill, he will loose one pound. If Neo originally weighs one pound, what is the minimum number of pills he must take to make his weight 2015 pounds.

3. (General Test P6)
   Consider all functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying

   $$f(f(x) + 2x + 20) = 15.$$

   Call an integer n *good* if $f(n)$ can take any integer value. In other words, if we fix n, for any integer m, there exists a function f such that $f(n) = m$. Find the sum of all good integers x.

4. (General Test P3)
   Let $ABCD$ be a quadrilateral with an inscribed circle ω that has center I. If $IA = 5$, $IB = 7$, $IC = 4$, $ID = 9$, find the value of $\frac{AB}{CD}$.

5. (Team Round P8)
   Find *any* quadruple of positive integers (a, b, c, d) satisfying $a^3 + b^4 + c^5 = d^{11}$ and $abc < 10^5$.

6. (Theme round P9)
   Consider a 9×9 grid of squares. Haruki fills each square in this grid with integer between 1 and 9, inclusive. The grid is called a *super-sudoku* if each of the following three conditions hold:

   - Each column in the grid contains each of the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ exactly once.

   - Each row in the grid contains each of the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ exactly once.

   - Every 3×3 sub square in the grid contains each of the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ exactly once.

   How many possible super-sudoku grids are there?