1.3   Season 1 Episode 3, 10/4/2015

1. [HMMT 2006, by Tiankai Liu] Six people, all of different weights, are trying to build a human pyramid: that is, they get into the formation shown on the right.

    We say that someone not in the bottom row is *supported by* each of the two closest people beneath her or him. How many different pyramids are possible, if nobody can be supported by anybody of lower weight?

    ```
      A
     B C
    D E F
    ```

2. Given that $P = (5, 0)$, $Q = (8, 21)$, and $R = (0, 15)$, show that one of the angles in triangle PQR has a measure of 45 degrees. (Trigonometry method shall not be used in your solution.)

3. Suppose that a quadrilateral is measured and found to have one of the following set of properties. Is this enough evidence to conclude that the quadrilateral is a parallelogram? Explain.

    (a) two pairs of equal nonadjacent sides

    (b) two pairs of equal nonadjacent angles

    (c) a pair of equal nonadjacent sides and a pair of equal nonadjacent angles

4. One can dissect a 5×5 chessboard into a few pieces such that these pieces can be reassembled to form a 3×3 chessboard and a 4×4 chessboard (so the fields of the chessboard are preserved) via *translations*. Achieve this task with as few pieces as possible.

5. [MathCounts 2015] Let $(\mathtt{A}, \mathtt{B}, \ldots, \mathtt{O})$ be a permutation of $(1, 2, \ldots, 15)$. These letters form a pyramid shown on the right.

    We say that some letter not in the bottom row is *supported by* each of the two closest letters beneath it. How many different permutations are possible, if no letter can be supported by a letter with a smaller numerical value?

    ```
          A
       B     C
     D   E  F   G
    H I  J K  L M  N O
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