Chapter 1

Season 1, Fall 2015 to Spring 2016

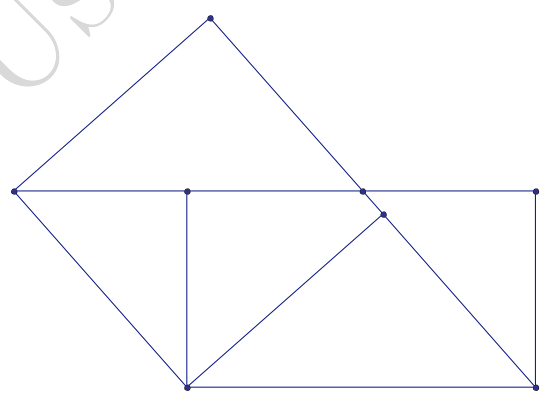
1.1 Season 1 Episode 1, 09/20/2015

1. A quadrilateral with *exactly one* pair of parallel sides is called a *trapezoid*. Why do we emphasize "exactly one" in this definition? Why a parallelogram is not considered as the special case of a trapezoid? There are many critical math ideas behind the answers to these questions and we will explain a few of them. To start, consider the following basic facts.

   (a) Prove that there are two pairs of congruent (base) angles in an isosceles trapezoid are congruent. (There is popular approach to prove this fact involving dissecting a trapezoid into a rectangle and two right triangles. Can you find the issues with this approach?)

   (b) The converse question: If a trapezoid has two pairs of equal adjacent angles, is it necessary that its non-parallel sides have the same length? Explain.

2. The diagram shown on the right provides a hint to dissect a 9×16 rectangle into finitely many pieces that can be reassembled into a square. Can you explain the details of this method? Does this method has any limitations (with respect to the dimensions of the rectangle)? Can you find other ways to do so? In particular, do it with as few pieces as possible.



3. The following problem was given in the 2012 MathCounts National:

   Quadrilateral $KLMN$ is inscribed in circle ω. The midpoint O of diagonal NL is also the center of ω. Diagonal KM meets segment OL in P. Given that $PL = 8$, $OP = 2$, $KN = 18$, and $PK = 9$, find the length of segment MN.

   Explain why the problem statement was wrong.

4. [AIME1 2004, by Harold Reiter] An integer is called *snakelike* if its decimal representation $a_1a_2a_3\dots a_k$ satisfies $a_i < a_{i+1}$ if i is odd and $a_i > a_{i+1}$ if i is even. How many snakelike integers between 1000 and 9999 have four distinct digits?

5. [AIME2 2007, by Zuming Feng] An integer is called *parity-monotonic* if its decimal representation $a_1a_2a_3\dots a_k$ satisfies $a_i < a_{i+1}$ if a_i is odd, and $a_i > a_{i+1}$ if a_i is even. How many four-digit parity-monotonic integers are there?