

Lectures on Challenging Mathematics

UC2 Geometry An Invitation to Computational Mathematics

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“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

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Contents

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1	Geometry Knowledge	3
1.1	Practices with geometric computations (part 1)	3
1.2	Area and lengths of the altitudes of a triangle in a coordinate plane	4
1.3	The sector	5
1.4	Practices with geometric computations (part 2)	6
1.5	Arcs and angles (part 1)	7
1.6	Arcs and angles (part 2)	9
1.7	The circumcenter of a triangle	10
1.8	Similarity (part 1)	11
1.9	The centroid of a triangle	12
1.10	Similarity (part 2)	13
1.11	The circumcenter and the centroid of a triangle	14
1.12	Circles (part 1)	15
1.13	The incenter and excenters of a triangle	16
1.14	The orthocenter of a triangle (part 1)	17
1.15	Circles (part 2)	18
1.16	The orthocenter of a triangle (part 2)	19
1.17	Angle-bisector theorem	20
1.18	Producing similar triangles	21
1.19	Centers of an isosceles triangle	22
1.20	Properties of the incircle and the angle bisector	23
2	Geometry Practices	25
2.1	Polygons and special angles (part 1)	25
2.2	Tangent circles and tangents to the circles (part 1)	26
2.3	Tangent circles and tangents to the circles (part 2)	27
2.4	Revisiting similarity (part 1)	28
2.5	Revisiting similarity (part 2)	29
3	Geometry Challenges	31
3.1	Polygons and special angles (part 2)	31
3.2	Practices with geometric computations (part 3)	32

3.3	Practices with geometric computations (part 4)	33
3.4	Practices with geometric computations (part 5)	34
3.5	Geometry problems from AMC10/12 and AIME (part 1)	35
4	Geometry Supplement	37
4.1	Folding and unfolding (part 1)	37
4.2	Tessellations and tilings (part 1)	39
4.3	Folding and unfolding (part 2)	40
4.4	Tessellations and tilings (part 2)	41
4.5	Geometry problems from AMC10/12 and AIME (part 2)	43