

Lectures on Challenging Mathematics

MO1M2

Elements of Olympiad Mathematics

Module 2

Winter 2017

Zuming Feng

Phillips Exeter Academy and IDEA Math

zfeng@exeter.edu

©Copyright 2008 – 2017 Idea Math

“Cogito ergo Sum” – “I think, therefore I am”

René Descartes (1596-1650)

©Copyright 2008 – 2017 Idea Math

Idea Math

Internal Use

Contents

©Copyright 2008 – 2017 Idea Math

1	Geometry	1
1.1	Challenges in geometry computations (part 1)	1
1.2	Obvious but not easy to prove facts about circles	2
1.3	The first look at the Euler line of a triangle	3
1.4	Angle chasing and the centers of triangles (part 1)	4
1.5	Challenges in geometry computations (part 2)	5
1.6	The Euler line	6
1.7	Folding, unfolding, and 3-D vision	7
1.8	Angle chasing and the centers of triangles (part 2)	8
1.9	Challenges in geometry computations (part 3)	9
1.10	Power-of-a-point theorem and the first look of the radical axis	10
1.11	Challenges in geometry computations (part 4)	11
1.12	Angle chasing and the centers of triangles (part 3)	12
1.13	Challenges in geometry computations (part 5)	13
1.14	Angle chasing and the centers of triangles (part 4)	14
1.15	Challenges in geometry computations (part 6)	15
1.16	Ceva's theorem and Menalaus' theorem (part 1)	16
1.17	Angle chasing and the centers of triangles (part 5)	17
1.18	Challenges in geometry computations (part 7)	18
1.19	Ceva's theorem and Menalaus' theorem (part 2)	19
1.20	Angle chasing and the centers of triangles (part 6)	20
2	Geometry Challenges	21
2.1	Tessellations and tiles (part 1)	21
2.2	Compass and straightedge constructions (part 1)	23
2.3	Area of a triangle in the coordinate plane	24
2.4	Tessellations and tiles (part 2)	25
2.5	Compass and straightedge constructions (part 2)	26
3	Number Theory	27
3.1	G.C.D., L.C.M., and prime factorization (part 1)	27
3.2	Number theory practice set 1	28

3.3	Floor and ceiling functions (part 1)	29
3.4	Number theory practice set 2	30
3.5	Floor and ceiling functions (part 2)	31
3.6	Practice test 1	32
3.7	Modular arithmetic (part 1)	33
3.8	Establishing the Frobenius Coin theorem (part 1)	34
3.9	Modular arithmetic (part 2)	35
3.10	Establishing the Frobenius Coin theorem (part 2)	36
3.11	Modular arithmetic (part 3)	37
3.12	Euler's totient function $\varphi(n)$	38
3.13	Number theory practice set 4	39
3.14	Perfect number, Mersenne prime, and the sum of divisors	40
3.15	Number theory practice set 5	41
3.16	Diophantine equations (part 1)	42
3.17	Modular arithmetic (part 4)	43
3.18	Diophantine equations (part 2)	44
3.19	Modular arithmetic (part 5)	45
3.20	Diophantine equations (part 3)	46
3.21	Modular arithmetic (part 6)	47
3.22	Number theory practice set 6	48
3.23	Number theory practice set 7	49
3.24	A direct proof of the Fundamental Theorem of Arithmetic	50
3.25	Number theory practice set 8	51
3.26	Number theory practice set 9	52
3.27	Number theory practice set 10	53
3.28	Number theory practice set 11	54
3.29	Number theory practice set 12	55
4	Number Theory Challenges	57
4.1	The first look at the Frobenius Coin theorem (Chicken McNugget theorem)	57